Quiz Six Solutions

1 The first thing to notice is that $\dfrac{\log_6(25)}{\log_6(e)} = \log_e(25) = \ln(25)$. Then

$$\begin{aligned} \ln(5^3) - \frac{\log_6(25)}{\log_6(e)} &= \ln(5^3) - \ln(25) \\ &= \ln(5^3/25) \\ &= \ln(5). \end{aligned}$$

This is as simple as possible.

2 We raise 2 to both sides of $\log_2(x^2 + x) = 2$ to get

$$2^{\log_2(x^2+x)} = 2^2,$$

or

$$x^2 + x = 4.$$

This simplifies to $x^2 + x - 4 = 0$, which has solutions

$$x = \frac{-1 \pm \sqrt{17}}{2}$$

by the quadratic formula.

3 The *half-life* of an element is the time it takes to decay to half the initial amount. In two half-lives, the element decays to half of half, or one quarter, the initial amount. Thus 3 hours is two half-lives, or one half-life is 1.5 hours.

4 The exponential model we'll use is $A(t) = A_0 b^t$, where $A(t)$ is the amount of 120-ium after t hours and $A_0 = A(0)$ is the initial amount. We know, then, that $A_0 = 15$ grams, and that $A(1.5) = \frac{1}{2}A_0$ (since the half-life is 1.5 hours). Thus $A_0 b^{1.5} = \frac{1}{2}A_0$, or $b^{1.5} = 1/2$. Taking the 1.5th root, we get $b = (1/2)^{1/1.5} = \sqrt[1.5]{1/2}$. (Notice that $1.5 = 3/2$, so really $\sqrt[1.5]{1/2} = \sqrt{(1/2)^3} = \sqrt{1/8}$ or $\frac{1}{2\sqrt{2}}$.) Plugging this in, we get

$$A(t) = 15\left(\frac{1}{2}\right)^{t/1.5}.$$