## Math 120 A - Spring 2009

Final Exam
June 6, 2009

Name: $\qquad$ Student ID no. : $\qquad$

## Signature:

$\qquad$ Section: $\qquad$

| 1 | 10 |  |
| :---: | :---: | :--- |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| Total | 70 |  |

- Answer all seven questions.
- You may use a scientific calculator during this examination. Graphing calculators are not allowed. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- If you use a trial-and-error or guess-and-check method, when an algebraic method is available, you will not receive full credit.
- You may use one hand-written 8.5 by 11 inch page of notes. Write your name on your notesheet and turn it in with your exam.
- Show all work for full credit.
- You have 170 minutes to complete the exam.

1. In the year 1995, the city of Arlot had a population of 12,000 . The population of Arlot doubles every 9 years.
The population of the city of Belb is growing exponentially. In the year 1990, there were twice as many people in the city of Belb as in the city of Arlot. In the year 2010, there will be twice as many people in Arlot as in Belb.
How long does it take the population of the city of Belb to triple?
2. Nathan and Adelaide are moving in the $x y$-plane. They both start moving at the same time, $t=0$.
At any time $t \geq 0$, Adelaide's location is given by the equations

$$
x=5-2 t, y=2+3 t
$$

with the coordinates in meters.
Nathan's location at time $t=0$ is the point $(10,1)$ and he moves along the line $y=$ $-0.5 x+6$, reaching the $y$-axis after 2 seconds.
(a) Give equations describing Nathan's $x$ - and $y$-coordinates at time $t$.
(b) Find the time $t$ when Nathan and Adelaide will be closest together.
3. The Circular Forest is in the shape of a perfect circle, with radius 12 km .

Benny walks in a straight line from a point 15 km west and 20 km south of the center of the forest. He walks to a point 1 km east and 16 km north of the center of the forest.
When Benny enters the forest, how far west of the forest's center is he?
4. Agatha wants to build a rectangular enclosure with four partitions, creating five separate holding areas for her llamas. An example of a possible shape for her enclosure is shown below.


Agatha has 500 feet of fencing to build the enclosure. The fencing will be used to create the enclosure and the partitions.
What should the dimensions of the enclosure be to maximize the area of the enclosure?
5. Harry is running around a circular track. From his starting point, it takes him 12 seconds to reach the easternmost point of the track. From his starting point, it takes him 35 seconds to reach the southernmost point of the track. The track has a radius of 100 meters.
(a) Using the center of the track as the origin of your coordinate system, express Harry's $x$ - and $y$-coordinates as functions of the time, $t$, since he started running.
(b) When Harry starts running, Sarah starts running counterclockwise from the northernmost point of the track. She runs with an angular speed of 0.13 radians per second. How long has Sarah been running when she passes Harry for the first time?
6. A spring is oscillating so that its length is a sinusoidal function of time. Its length varies from a minimum of 10 cm to a maximum of 14 cm . At time $t=0$ seconds, the length of the spring was 12 cm , and it was decreasing in length. It then reached a minimum length at time $t=1.2$ seconds.
Between time $t=0$ and $t=8$ seconds, how much of the time was the spring longer than 13.5 cm ?
7. Let $g(x)=\frac{1}{x}$ and $f(x)=3 x-2$. A fixed point of a function $k(x)$ is any number $p$ so that $k(p)=p$.
(a) Let $j(x)=f(x)-g(x)$. Find the fixed points of $j(x)$.
(b) Let $h(x)=g(f(x))$. Find $h^{-1}(x)$.

