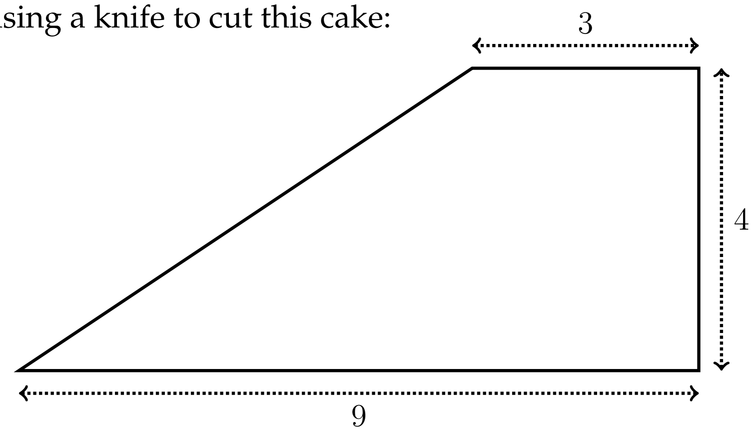
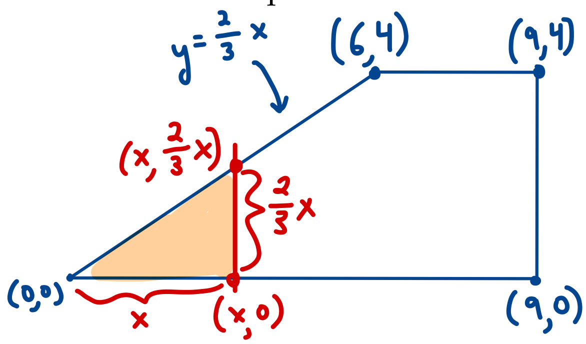
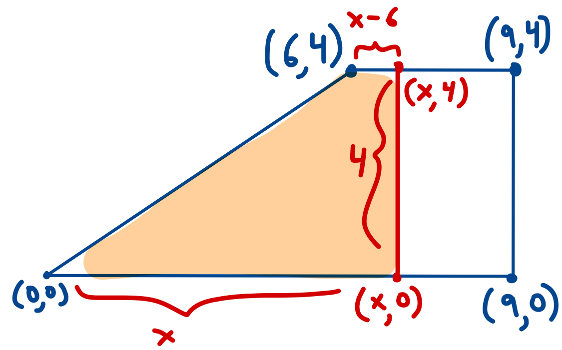
1. Mrs. White is in the dining room using a knife to cut this cake:



(a) **[6 points]** Suppose she makes a vertical cut x units from the left end of the cake. Write a multipart function for the area to the left of the cut.





$A = \frac{1}{2}(x)\left(\frac{2}{3}x\right) = \frac{1}{3}x^2$

$A = \frac{1}{2}(x + x - 6)(4) = 4x - 12$

$$A(x) = \begin{cases} \frac{1}{3}x^2 & \text{if } 0 \leq x \leq 6 \\ 4x - 12 & \text{if } 6 \leq x \leq 9 \end{cases}$$

(b) **[4 points]** Mrs. White wants to make two vertical cuts to divide the area of the cake into three pieces of equal area. How far in should she make those two cuts?

Total area of cake $\frac{1}{2}(9+3)(4) = 24$

Want to cut into $\frac{1}{3}$s. $A(x) = 8$, $A(x) = 16$

$A(x) = 8$

$0 \leq x \leq 6$: $\frac{1}{3}x^2 = 8$, $x^2 = 24$, $\boxed{x = \sqrt{24}}$

$6 \leq x \leq 9$: $4x - 12 = 8$, $\cancel{x = 5}$

and

$A(x) = 16$

$0 \leq x \leq 6$: $\frac{1}{3}x^2 = 16$, $x^2 = 48$, $\cancel{x = \sqrt{48}}$

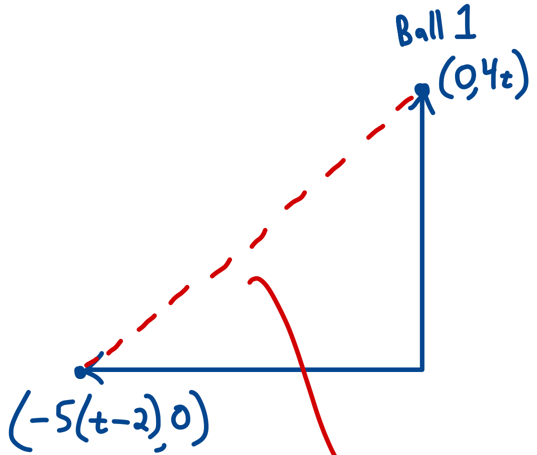
$6 \leq x \leq 9$: $4x - 12 = 16$, $\boxed{x = 7}$

2. **[10 points]** Colonel Mustard is in the billiard room, and has tied two billiard balls together with an 80-inch rope.

   At time $t = 0$, he knocks the first ball north at a constant speed of 4 inches per second.

   Two seconds later, he knocks the second ball from the same starting position as the first ball. It travels west at a constant speed of 5 inches per second.

   When does the rope become tight?



$$\text{Dist} = \sqrt{(4t)^2 + (5(t-2))^2} = 80$$

$$16t^2 + 25t^2 - 100t + 100 = 6400$$

$$41t^2 - 100t - 6300 = 0$$

$$t = \frac{100 \pm \sqrt{100^2 - 4(41)(-6300)}}{2(41)}$$

$$\approx \boxed{13.675 \text{ seconds}}$$

3. Professor Plum is investigating a leaky lead pipe in the conservatory. It seems to be causing the flowers to grow exponentially!

(a) **[3 points]** The number of forsythias doubles every 25 days.

Initially, there were 20 forsythias.

Write a function $f(t)$ for the number of forsythias after t days.

$$f(t) = A_0 b^t$$

$$A_0 = 20$$

$$b^{25} = 2 \rightarrow b = 2^{1/25}$$

$$\boxed{f(t) = 20 \cdot 2^{\frac{t}{25}}}$$

(b) **[4 points]** The number of geraniums is also growing exponentially.

After 16 days, there were 10 geraniums, and after 22 days, there were 13 geraniums.

Write a function $g(t)$ for the number of geraniums after t days.

$$g(t) = A_0 b^t$$

$$10 = A_0 b^{16}$$

$$13 = A_0 b^{22}$$

$\div$

$$1.3 = b^6 \rightarrow b \approx 1.0447 \rightarrow A_0 = \frac{10}{b^{16}} \approx 4.97$$

$$\boxed{g(t) = 4.97(1.0447)^t}$$

(c) **[3 points]** When will the number of forsythias equal the number of geraniums?

Round your answer to the nearest day.

$$20 \cdot 2^{\frac{t}{25}} = 4.97(1.0447)^t$$

$\downarrow \ln(\ )$

$$\ln\left(20 \cdot 2^{\frac{t}{25}}\right) = \ln\left(4.97(1.0447)^t\right)$$

$$\ln(20) + \frac{t}{25}\ln(2) = \ln(4.97) + t\ln(1.0447)$$

$$t = \frac{\ln(20) - \ln(4.97)}{\ln(1.0447) - \frac{1}{25}\ln(2)} \approx \boxed{87 \text{ days}}$$

4. Mrs. Peacock is standing in the study, where a candlestick is positioned on the floor.

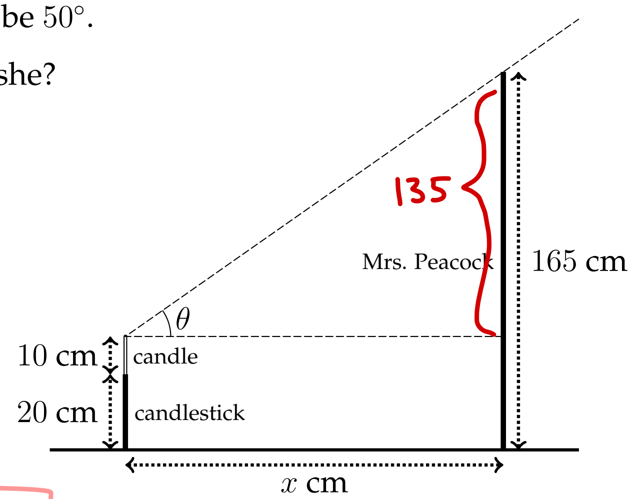
Mrs. Peacock is 165 centimeters tall. The candlestick is 20 centimeters tall, and it's holding a candle which is 10 centimeters tall.

Let θ be the angle of elevation of Mrs. Peacock's head relative to the top of the candle, as shown in the picture below.

(a) **[5 points]** Mrs. Peacock measures θ to be $50°$.

How far away from the candlestick is she?

(In other words, what's x?)



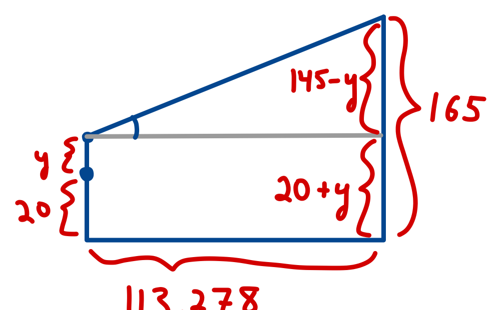
$$\tan(50°) = \frac{135}{x}$$

$$x = \frac{135}{\tan(50°)} \approx \boxed{113.278 \text{ cm}}$$

(b) **[5 points]** The candle burns at a constant speed. After 1 minute, the angle θ is $51°$.

When will the candle burn all the way down?

$$\tan(51) = \frac{145-y}{113.278}$$



$$y = 145 - 113.278\tan(51)$$

$$\approx 5.113 \text{ cm}$$

So the candle burns at a rate of 4.887 cm/min, which means it takes a total of $\frac{10}{4.887} \approx \boxed{2.046 \text{ min}}$ to burn

(or $\boxed{122.77 \text{ seconds}}$)

5. Miss Scarlett is in the ballroom, dancing to a Beatles album.

Her dance proceeds clockwise in a circle of radius 20 feet at a constant speed. It takes her 17 seconds to make one complete lap, and she reaches the northernmost point 5 seconds after she starts.

(a) **[3 points]** Find Miss Scarlett's linear speed.

$$\omega = \frac{2\pi}{17}$$

$$r = 20$$

$$v = \omega r = \boxed{\frac{40\pi}{17} \text{ ft/sec}}$$

(b) **[4 points]** Impose a coordinate system with the center of the circle at the origin.

Write parametric equations for Miss Scarlett's coordinates after t seconds.

$$\theta_0 - 5\omega = \frac{\pi}{2} \rightarrow \theta_0 = \frac{\pi}{2} + \frac{10\pi}{17} = \frac{37\pi}{34}$$

$$\boxed{\begin{aligned} x &= 20\cos\left(\frac{37\pi}{34} - \frac{2\pi}{17}t\right) \\ y &= 20\sin\left(\frac{37\pi}{34} - \frac{2\pi}{17}t\right) \end{aligned}}$$

(c) **[3 points]** After 35 **minutes**, how far **east** is Miss Scarlett from her starting point?

Start $\quad x = 20\cos\left(\frac{37\pi}{34}\right) \approx -19.2365$

$\underbrace{\text{35 minutes}}_{\text{2100 sec}} \quad x = 20\cos\left(\frac{37\pi}{34} - \frac{2\pi}{17}(2100)\right) \approx 19.9147$

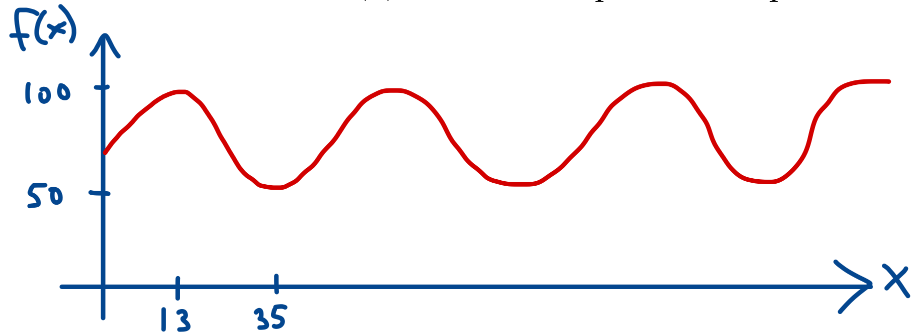
$$19.9147 - (-19.2365) \approx \boxed{39.1512 \text{ ft}} \text{ east}$$

6. **[5 points per part]** Mr. Green is in the kitchen, using a wrench to adjust the water pressure under the sink. The pressure is a sinusoidal function of time.

The pressure first reaches its maximum of 100 psi 13 minutes after the start.

It then decreases, reaching a minimum of 50 psi 35 minutes after the start.

(a) Find a function $f(x)$ for the water pressure (in psi) x minutes after the start.



$A = 25$

$B = (35-13)\times 2 = 44$

$C = 13 - 11 = 2$

$D = 75$

$$\boxed{f(x) = 25\sin\left(\frac{2\pi}{44}(x-2)\right) + 75}$$

(b) The maximum recommended water pressure in a home is 80 psi. In the first hour, for how much time (total) is the pressure above this level?

$$25\sin\left(\frac{2\pi}{44}(x-2)\right) + 75 = 80$$

$$\sin\left(\frac{2\pi}{44}(x-2)\right) = \frac{1}{5}$$

$$\frac{2\pi}{44}(x-2) = \sin^{-1}\left(\frac{1}{5}\right)$$

$$x = 2 + \frac{44}{2\pi}\sin^{-1}\left(\frac{1}{5}\right) \approx 3.41$$

↑ principal solution

Symmetry solution

$4 + 22 - 3.41 = 22.59$

List of solutions

3.41

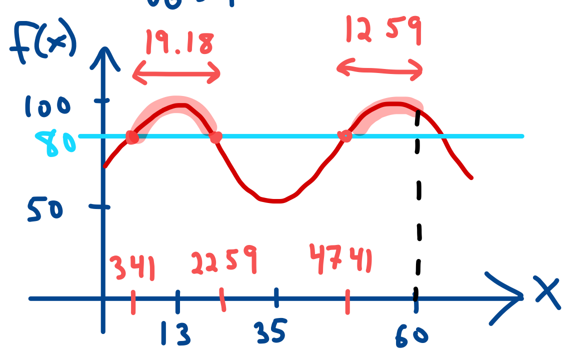
22 59

47 41

66 59

+44

+44



Total: $19.18 + 12.59 = \boxed{31.77 \text{ minutes}}$

7. Mr. Boddy is in the library with a linear-to-linear rational function:

$$f(x) = \frac{3x+2}{x+4}$$

(a) **[4 points]** Find the following data about this function:

- Horizontal asymptote: $a=3$ $\boxed{y=3}$
- Vertical asymptote: $d=4$ $\boxed{x=-4}$
- x-intercept: $3x+2=0 \rightarrow x=\frac{-2}{3} \rightarrow \left(\frac{-2}{3}, 0\right)$
- y-intercept: $x=0 \rightarrow y=\frac{1}{2} \rightarrow \left(0, \frac{1}{2}\right)$

(b) **[3 points]** Compute $f(f(6))$.

$$f(f(6)) = f\left(\frac{3\cdot 6+2}{6+4}\right) = f(2) = \frac{3\cdot 2+2}{2+4} = \boxed{\frac{4}{3}}$$

(c) **[3 points]** Write a formula for $f^{-1}(x)$.

$$y = \frac{3x+2}{x+4}$$

$$y(x+4) = 3x+2$$

$$xy + 4y = 3x+2$$

$$xy - 3x = 2-4y$$

$$x(y-3) = 2-4y$$

$$x = \frac{2-4y}{y-3}$$

$$\boxed{f^{-1}(x) = \frac{2-4x}{x-3}}$$