Math 120

Midterm Exam 2

Name \_\_\_\_\_

Directions: Please show all of your work. Full credit will only be given when work is clearly presented. Use appropriate units and labels.

1a. (8 points) If  $\sin \theta = \frac{-9}{15}$ , then what are two possible values for  $\cos \theta$ ?

1b. (12 points) Aaron is hiking when he notices a bird on the top of a tree. He notes that the angle between the horizontal and his line of sight to the treetop is  $15^{\circ}$ . He walks on level ground at 3  $\frac{feet}{sec}$  toward the base of the tree. Ten seconds later, the angle between the horizontal and his line of sight is  $25^{\circ}$ . If Aaron's eyes are 5.5 feet off the ground, what is the height of the tree?

- 2. Let  $f(x) = 3 \sqrt{4x 20}$  on the domain  $x \ge 5$ . a. (10 points) Find  $f^{-1}(x)$ .

b. (10 points) What are the domain and range of  $f^{-1}(x)$ ?

c. (5 points) What is  $f(f^{-1}(x))$ ?

3. While on his hiking expedition, Aaron found a rope swing dangling 6 feet above a river. He climbed a nearby tree and began swinging. At the highest point of the arc of his swing, he was 22 feet above the surface of the river. He starts his stopwatch and notices that he is at his lowest point when his watch reads 2 seconds and again when it reads 8 seconds.
a. (10 points) Find a sinusiodal function $A(t)$ giving Aaron's height over the river surface in terms of time ( $t = 0$ when Aaron starts his stopwatch).

b. (10 points) Sketch a graph of A(t). Be sure to graph at least one period and label the significant features of your graph.

4. Continuing his hiking adventures, Aaron surprisingly stumbles upon a lake shaped perfectly like a circle. After naming it Lake Amazing, Aaron decides to walk around the lake. He walks clockwise at a rate of 3 $\frac{feet}{sec}$ keeping 3 feet from the edge of the lake. His walk takes him half an hour. (be careful of units)
a. (5 points) What is Aaron's angular velocity in $\frac{rads}{min}$ ?
b. (8 points) What is the radius of Lake Amazing?
c. (12 points) Aaron started his walk at the southernmost point of Lake Amazing. If you impose a coordinate system with the origin at the center of Lake Amazing, what are Aaron's x-coordinate and y-coordinate after 12 minutes?
d. (10 points) More surprisingly still, Anna has already found Lake Amazing and is canoeing counter-clockwise, keeping a distance of 100 feet from shore. She is at the northernmost point when Aaron begins his walk and her angular velocity is $0.56 \frac{rads}{min}$ . When do Aaron and Anna pass?