Math 120A
Summer, 2001
Final 2

Date: Aug 17, 2001   Name ▬▬▬▬▬▬▬▬▬▬▬

- There are 5 problems of 10 points each, for a total of 50 points.

- You may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes (no photocopied or printed material). You should also have a clear plastic rular.

- *Show your work:* If your work demonstrates that you had the right idea, you might get partial credit even if your answer is wrong. On the other hand, if your answer is not justified by your work, you might lose points even if your answer is correct. If you need more room, continue your work on the back of the page.

- Leave your answers in exact form (for example, $\sqrt{2}$ rather than 1.414213562...). If you do give decimal answers, round your answers to two (2) decimal places, unless otherwise specified. Calculators are not allowed.

- If any particular question appears to be 'tough' move on to the next. Do not waste time over one single problem.

- If you have a question, raise your hand and Kelly or I will come to your seat.

- Good luck!

For grader's use only

Problem	Points
1	
2	
3	
4	
5	
Total	

1. Warming up:

(a)

x	0	1	2	3	4	5
$f(x)$	5	2	3	0	1	4
$g(x)$	2	0	5	4	3	1

Complete the following table (some of them might not be defined): (5 points)

Function	Expand	Evaluate
$(f+g)(5)$	$f(5)+g(5)$	$4+1=5$
$(f-g)(3)$		
$(fg)(2)$		
$\dfrac{f}{g}(1)$		
$f^{-1}(4)$		
$g(6)$		

(b) State whether the following are **true** or **false**: (3 points)

  i. $f(x) = x^2 - 4, -1 < x < 1$ is one-one.

  ii. $f(x) = \ln(x)$ is onto.

  iii. A pace of 2.5 $min/mile$ is faster than 20 mph.

(c) f is a rational function. Pick the right answer to each question from the following: (2 points)
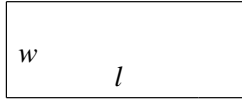
zeroes of f horizontal asymptotes of f
where f crosses the y-axis   vertical asumptotes of f

  i. Vertical asymptote(s) of f^{-1} are

  ii. Vertical asymptote(s) of $\dfrac{1}{f}$ are

2. Graham has a 24 ft long fence. He wants to surround a rectangular plot of land with that. All lengths in this problem are in ft and all area in $sqft$.

(a) Write the length l of the plot as a function of the width w. (1 point)

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┌──────────────┐
│ w            │
│      l       │
└──────────────┘
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(b) Write the area A as a function of the width w. What are the domain and range of this function? (3 points)

(c) What is the maximum possible area that can be surrounded by this fence? When does it happen (i.e. for what w)? (2 points)

(d) Ignoring units, draw the graphs of both the functions you got in a) and b) on the same axes. Label the zeroes and the x-coordinate for the maximum area. (2 points)

(e) Where do the graphs intersect each other? Give both the coordinates. (2 points)

3. $f(t) = e^t$, $g(t) = 5(\frac{1}{2})^t$, get the graph of $y = g(t)$ from that of $y = f(t)$ using **only** horizontal shift, horizontal flip and horizontal dilation. You may choose whatever order you want. But mention each step carefully. At the end write g in terms of f. (10 points)

Step	"What"(circle one)	"How much"
1	shift / dilation / flip	
2	shift / dilation / flip	
3	shift / dilation / flip	

4. Below is the graph of $y = p(x)$ where p is a polynomial.

(a) Is the degree of the polynimial even or odd? If p is of the form $a - nx^n + a_{n-1}x^{n-1} + ... + a_0$ then what is the 'sign' of a_n? What are the roots of p? Is p an 'odd' or 'even' function or none? (3 points)
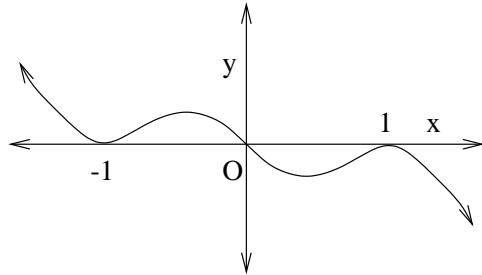

(b) Complete the following table: (5 points)

x-values (intervals)	$p(x)$
	positive
	negetive
	increasing
	decreasing

(c) Factorise $x^5 + x^4 - 9x^3 - 9x^2$ into linear factors (this can be done!). (2 points)

5. Tinni was maikng a birthday card for her friend. After finishing she enclosed it in an envelope and sealed it with a round sticker of diameter $2\ cm$. The triangular flap of the envelope makes a $90°$ angle at the tip.

(a) Impose a coordinate system to this picture of the above situation such that the $y - axis$ is the axis of symmetry. Mark the origin, axes and the points $(1, 0)$ and $(0, 1)$. (2 points)
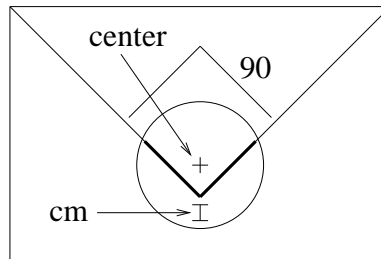
(b) What is the equation of the circle made by the edge of the sticker? (2 points)

(c) What are the equations of the two lines made the edges of the flap? (2 points)

(d) Find out the total length l of the thickened edge of the flap that is covered by the sticker. [**Hint:** You need to find where the circle intersects one of the lines, the distance of it from the corner and then use symmetry!] (4 points)