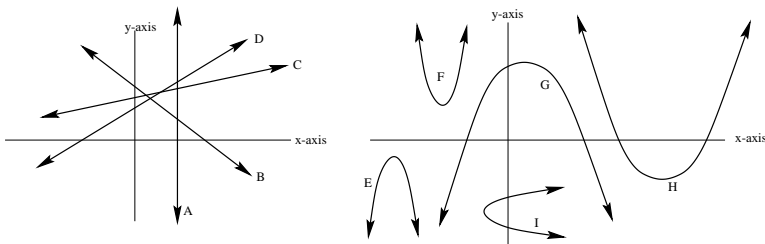


TA: _____

There were two versions of the midterm. The version depends on whether the point totals on the cover page are listed on the LEFT or RIGHT side. The answers below are for the version with the totals on the LEFT. The places where the two versions differ are noted with a ★ symbol.

1. (6 pts; 2 pts each) For each of the following, circle T or F to indicate if the statement is TRUE or FALSE. Circle only one answer.
 - (a) TRUE : The lines $y = 2x + 1$ and $y = -0.5x$ are perpendicular.
 (★ FALSE: The lines $y = 2x + 1$ and $y = 0.5x$ are perpendicular.)
 - (b) TRUE: If $g(x) = 3x - 1$, then $g(g(x)) = 9x - 4$.
 - (c) FALSE: To draw the graph of $f(x - 3)$, horizontally shift the graph of $f(x)$ to the left 3 units.
 (★TRUE: To draw the graph of $f(x + 3)$, horizontally shift the graph of $f(x)$ to the left 3 units.)

2. (3 pts) Below you will find nine curves labeled A, B, C, D, E, F, G, H, I.
 - (a) List all curves that are the graphs of increasing linear functions. Answer: C, D
 - (b) Suppose that $g(x)$ is a quadratic function with a maximum value on the domain of all real numbers. In addition, assume this maximum value is a negative number. List all curves that could represent the graph of $g(x)$. Answer: E.

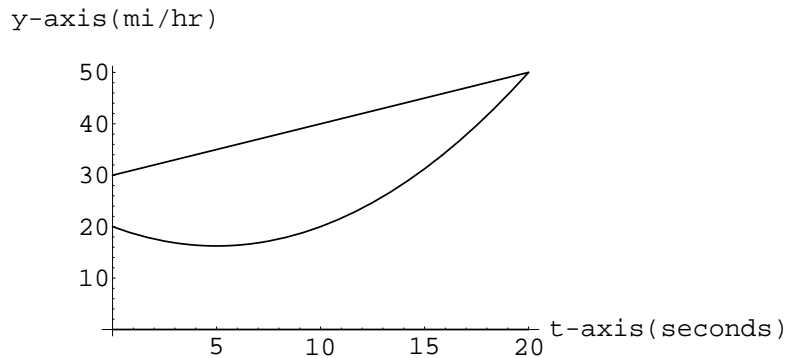


3. (5 pts) Let $f(x) = x^2$.
 - (a) Find $f(4 + h)$ and simplify. $f(4 + h) = (4 + h)^2 = 16 + 8h + h^2$.
 - (b) Find $\frac{f(4+h)-f(4)}{h}$ and simplify. $\frac{f(4+h)-f(4)}{h} = \frac{16+8h+h^2-4^2}{h} = \frac{8h+h^2}{h} = 8 + h$.
 ★ Other version, (a) $f(2+h) = (2+h)^2 = 4+4h+h^2$. (b) $\frac{f(2+h)-f(2)}{h} = \frac{4+4h+h^2-2^2}{h} = \frac{4h+h^2}{h} = 4 + h$.

4. (18 pts) A motorcycle and a car pass through an intersection at the same moment. The speed of the motorcycle after t seconds is given by the quadratic function

$$m(t) = \frac{3}{20}t^2 - \frac{3}{2}t + 20 \text{ mi/hr.}$$

The speed of the car is given by a linear function $c(t)$. Assume that the car is traveling 30 mi/hr when it enters the intersection and 50 mi/hr after 20 seconds. Both functions are graphed on the domain $0 \leq t \leq 20$ seconds.



- (a) (2 pts) FALSE : When both vehicles enter the intersection, the motorcycle is going faster.

[[★ TRUE: When both vehicles enter the intersection, the car is going faster.]]

- (b) (4 pts) Find the formula for $c(t)$. Compute $c(10)$.

By two point formula:

$$c(t) = \left(\frac{50 - 30}{20 - 0} \right) (t - 20) + 50 = t - 20 + 50 = t + 30$$

$$c(10) = 40.$$

[[★ $c(15) = 45$]]

- (c) (3 pts) Assume that Q is a point on the graph of $c(t)$ with y -coordinate 35. Label this point on the graph AND compute the t -coordinate of Q .

To get t coordinates, solve $35 = c(t)$ to get $t = 5$. $Q = (5, 35)$ should be plotted.

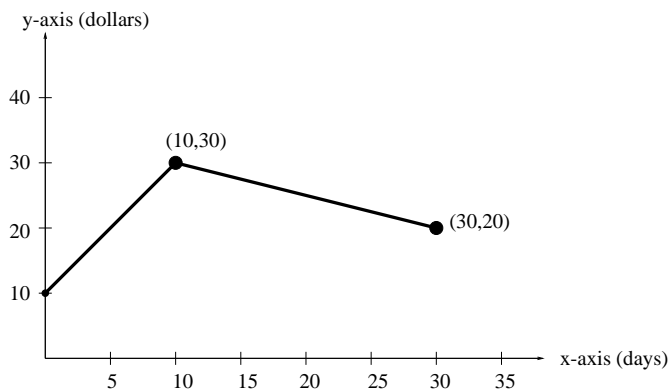
[[★ $Q = (10, 40)$]]

- (d) (9 pts) Find when (i.e. the time) the difference between the speed of the car and the speed of the motorcycle is the largest. What is the largest difference between the speed of the car and the speed of the motorcycle?

Form the new function $d(t) = c(t) - m(t) = -\frac{3}{20}t^2 + 2.5t + 10$. Put in vertex form: $y = d(t) = -\frac{3}{20}(t - 8.33)^2 + 20.4$. Conclude largest speed difference is 20.4 mph and it occurs at time $t = 8.33$ seconds.

5. (18 pts) You purchase stock in the company *Whizco*. On the day you buy the stock, the price is \$10 a share. After x days, the multipart function $y = f(x)$ calculates the price of one share. The graph of $f(x)$ is pictured on the domain $0 \leq x \leq 30$. The multipart rule is

$$f(x) = \begin{cases} 2x + 10 & \text{if } 0 \leq x \leq 10 \\ -\frac{1}{2}x + 35 & \text{if } 10 \leq x \leq 30 \end{cases}$$



- (a) (2 pts) What is the price of one share of stock after 12 days?
 Plug $t = 12$ into function using the second case of the multipart rule: $f(12) = 29$ dollars.
 [[★ $f(14) = 28$ dollars.]]
- (b) (4 pts) Determine when the share price is increasing AND the rate (dollars/day) it is increasing.

The graph is increasing between the points $(0, 10)$ and $(10, 30)$. This means the function is increasing on the domain $0 \leq x \leq 10$. It is ok if you wrote $0 \leq x < 10$, since we have not made a careful distinction of the graphical behavior at a local extrema.

- (c) (7 pts) The function $x = h(t) = \frac{t}{24}$ converts t hours into x days. (For example, if $t = 48$ hours, then $x = h(48) = 2$ days.) (i) Find the rule for the composition $f(h(t))$. (ii) Explain in a complete english sentence what the function $f(h(t))$ calculates. (iii) Explain how to use the construction tools discussed in class to obtain the graph of $f(h(t))$ from the graph of f .

(i) The multipart rule is given below; you replace “ x ” by “ $\frac{t}{24}$ ” everywhere in the rule for $f(x)$ and do the arithmetic to simplify. (ii) The function $f(h(t))$ computes the price of one share of stock after t hours. (iii) The function $f(h(t)) = f(\frac{t}{24})$ is a horizontal dilation of the graph of f with a dilation factor of 24; that means the graph gets horizontally stretched, since the dilation factor is bigger than one.

$$f(h(t)) = \begin{cases} \frac{t}{12} + 10 & \text{if } 0 \leq t \leq 240 \\ -\frac{t}{48} + 35 & \text{if } 240 \leq t \leq 720 \end{cases}$$

- (d) (5 pts) Determine when the price of one share is at least \$25.
 Need to solve the equation $25 = f(x)$. Must consider two cases separately. First, solve $25 = 2x + 10$ on the domain $0 \leq x \leq 10$; get $x = 7.5$. Next, solve $25 = -\frac{1}{2}x + 35$ on the domain $10 \leq x \leq 30$ to get $x = 20$. Conclude the share price is AT LEAST \$25 on the domain $7.5 \leq x \leq 20$ days.

[[★ To solve $22 = f(x)$, end up with interval: $6 \leq x \leq 26$]]