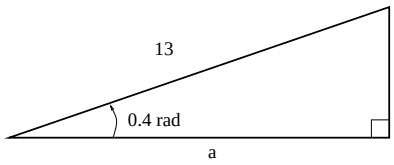
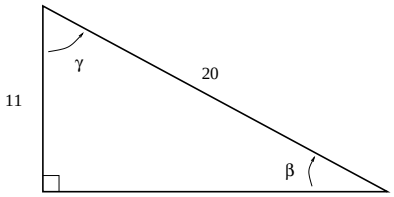
KEY TO MIDTERM #2, W1998

1. (10 pts) Short Answer problems:

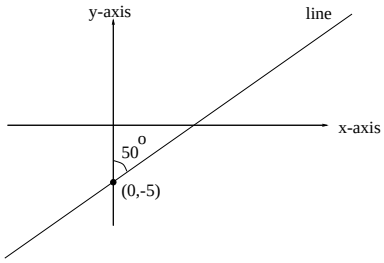
   (a) (2pts) Here is a right triangle. Find a.
   $\cos(0.4) = \frac{a}{13}$, so $a = 13\cos(0.4) = 11.97$.



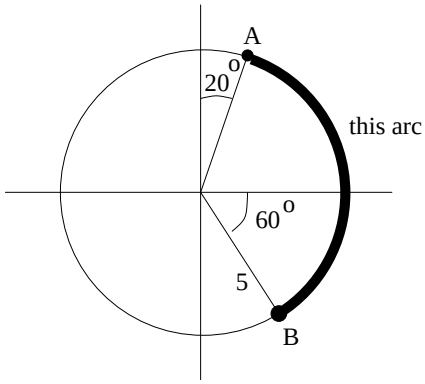
   (b) (3pts) Here is a right triangle. Find β and γ in radian units:
   $\beta = \sin^{-1}(\frac{11}{20}) = 0.582$ and $\gamma = \frac{\pi}{2} - \beta = 0.988$rad.



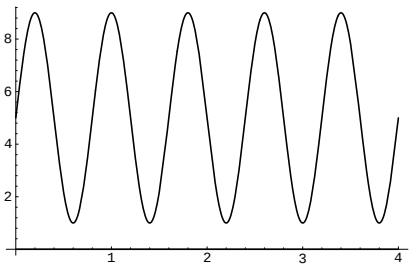
   (c) (2pts) Find the equation of the pictured line. Must use 40^o angle to calculate slope; NOT 50^o angle.
   slope line=$\tan(40^o)$, so $y = 0.84x - 5$.
   .

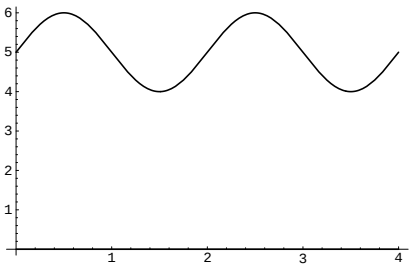


   (d) (2pts) Here is a circle of radius 5. What is the length of the arc between A and B?
   The arc is subtended by an angle of measure 130^o. Convert this to 2.27 radians. Now arc length formula gives:
   length=(2.27)5=11.34. .



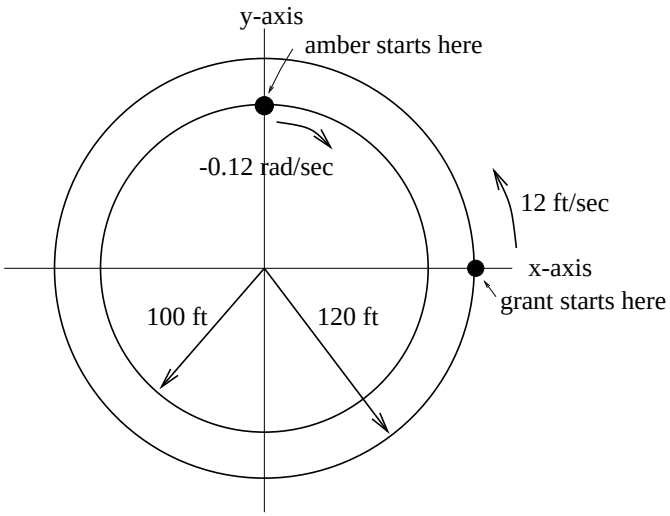
   (e) (1pts) Which function has the largest period?





   The "repeating" unit of the right graph is longer than that for the left graph; so the right graph has a larger period. (The amplitude of the left graph is largest, but that was not the question asked.)

2. (18 pts)Two runners are moving around a circular track. Grant runs around the outside lane counterclockwise, which is a circle of radius 120 feet and his linear speed is 12 ft/sec. Amber runs around the inside lane clockwise, which is a circle of radius 100 feet and her angular speed is - 0.12 rad/sec. This is a picture looking down from above with an imposed coordinate system. (RESET YOUR CALCULATOR TO RADIAN MODE!)



(a) (2pts) What is Grant's angular speed in units of "rad/sec"?
$\omega = 0.1$ rad/s.

(b) (6pts) Where is Grant located in 30 seconds (his xy-coordinates)? Mark this location in the picture.
location angle for grant is $\omega \times 30 = 3$ rad. So,
grant location=$(120\cos(3), 120\sin(3)) = (-118.8, 16.93)$.

(c) (8pts) Where is Amber located in 20 seconds (her xy-coordinates)? Mark this location in the picture.
location angle for amber=$\frac{\pi}{2} + (20)(-0.12) = -0.829$ rad.
amber location=$(100\cos(-0.829), 100\sin(-0.829)) = (67.56, -73.72)$.
(Alternate version used 16 seconds, so get -0.349 rad for location angle. location becomes $(100\cos(-0.349), 100\sin(-0.349)) = (93.97, -34.2)$.)

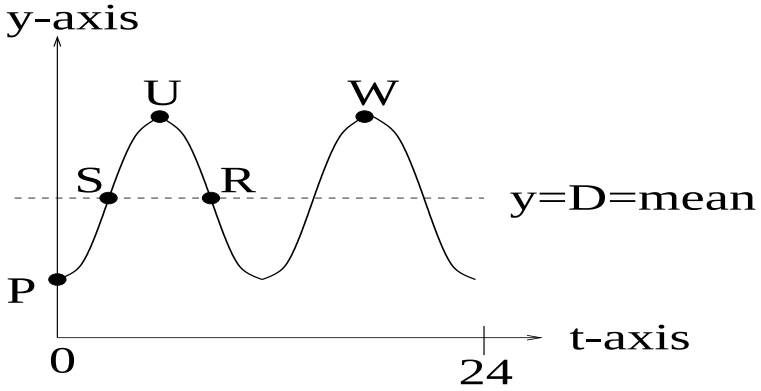
(d) (2pts) Find the time when Amber and Grant pass each other? (Note: They pass each other when they are both located on the same radial line.)
Angle between them in t seconds is $\theta(t) = \frac{\pi}{2} - 0.12t - 0.1t = \frac{\pi}{2} - 0.22t$. Need to know when $\theta(t) = 0$, get $t = 7.14$ seconds.

3. (22 pts) A $PCR\ machine$ is a biotech device that precisely changes the temperature of a small chamber containing DNA samples. Suppose the temperature ^oC in the chamber at time t minutes is given by the sinusoidal function:

$$y = d(t) = 34\sin(\frac{\pi}{6}t - \frac{\pi}{2}) + 60 = 34\sin(\frac{2\pi}{12}(t-3)) + 60.$$

(a) (4pts) Find the amplitude A, period B, phase shift C, and mean D:
$A = 34, B = 12, C = 3, D = 60$

(b) (8pts) Here is a graph of $y = d(t)$ on the domain $0 \leq t \leq 24$ minutes.



Make sure to justify your answer in terms of the data you calculated in (a).

i. Find the coordinates of S and R.
$S = (C, D) = (3, 60), R = (C + B/2, D) = (9, 60)$

ii. Find the coordinates of P.
$P = (0, 60 - A) = (0, 26)$.

iii. Find the coordinates of the maximum W.
$U = (C + B/4, D + A) = (6, 94)$, so $W = (6 + B, 94) = (18, 94)$.

(c) (8pts) During the first 24 minutes, find the <u>total</u> amount of time the DNA chamber has a temperature of <u>at least</u> 80^oC.

Need to solve $80 = 34\sin(\frac{2\pi}{12}(t-3)) + 60$; i.e.
$\frac{20}{34} = \sin(\frac{2\pi}{12}(t-3))$
To find principal sln, $sin^{-1}(\frac{20}{34}) = sin^{-1}\sin(\frac{2\pi}{12}(t-3))$; so $0.6289 = \frac{2\pi}{12}(t-3)$. Solve for t: $t = 4.2$ minutes. To find symmetry sln, $-sin^{-1}(\frac{20}{34}) + \pi = sin^{-1}\sin(\frac{2\pi}{12}(t-3))$; so $2.513 = \frac{2\pi}{12}(t-3)$. Solve for t: $t = 7.8$ minutes. During first 12 minutes, a total of (7.8-4.2) minutes at least 80^o. Double this to get 7.2 minutes total. (Alternate version used 70^o. You will get $t = 3.57, 8.43$ for principal and symmetry slns. Get $2(8.43 - 3.57) = 9.72$ minutes)

(d) (2pts) During the first 24 minutes, find the times when the temperature in the chamber is increasing.

Graph increases from P to U and from minimum after R up to W. this would be the time intervals $0 \leq t \leq 6$ and $12 \leq t \leq 18$. (Alternate version asked where decreasing, so get $6 \leq t \leq 12$ and $18 \leq t \leq 24$.)