Math 120A
January 15, 1998
Quiz #2 (20 points)

# QUIZ #2 KEY

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Instructions: You have 25 minutes. You **MUST** show work for credit. If in doubt, ask for clarification.

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1. (18pt) The value of a house in Seattle in the year $t$ is modeled by the linear function

$$s(t) = 7000(t - 1970) + 40000 \text{ dollars}$$

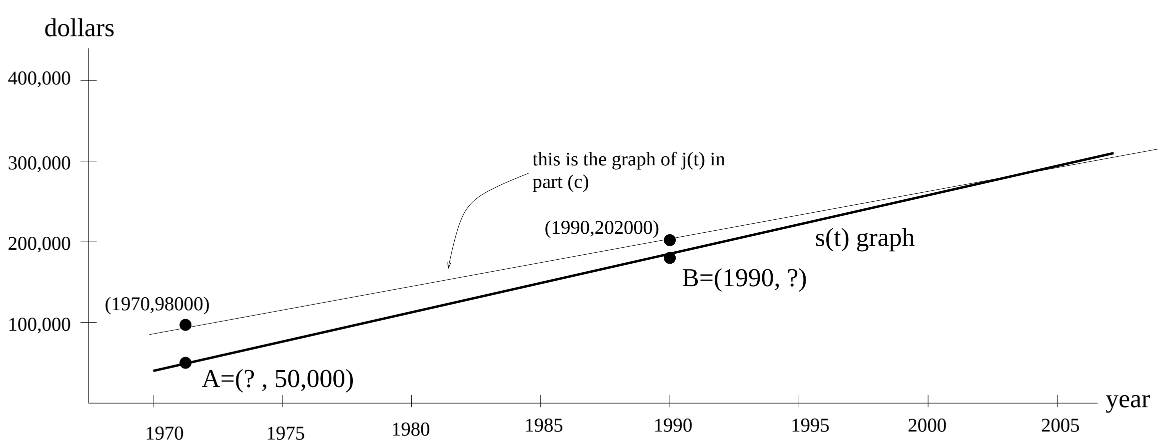
on the domain $1970 \leq t$.

On the other hand, in La Jolla (California), the value of a house was \$98,000 in 1970 and \$202,000 in 1990. Assume the value of the house in the year $t$ in La Jolla is modeled by a linear function $j(t)$ on the domain $1970 \leq t$.

(a) (2pt) What was the value of a house in Seattle in 1983? (Round to nearest dollar.)
Plug 1983 into the model for the Seattle house value:
$s(1983) = 7000(1983 - 1970) + 40000 = \$131,000;$

(b) (4pt) Here is a picture of the graph of $s(t)$. What are the coordinates of the two points labeled $A$ and $B$? (Round to one decimal place.)

Every point on the graph looks like $(t, s(t))$, so $A$ is a point where $s(t) = 50000$ is given to us and we need to find $t$. We have to solve this equation for $t$
$s(t) = 50000$ i.e., $7000(t-1970)+40000 = 50000$. You will get 1971.4. So, $A = (1971.4, 50000)$
If you know the $t$ coordinate of a point on the graph, then the $y$ coordinate is the function evaluated at that $t$ coordinate. For this example,
$B = (1990, s(1990)) = (1990, 7000(1990 - 1970) + 40000) = (1990, 180000)$.



(c) (5pt) Find the formula for $j(t)$. Sketch the graph of $j(t)$ in the coordinate system of part (b).

The problem gives us two data points on the line representing La Jolla house values: (1970,98000) and (1990,202000). Use the two point formula for a line:

$$\begin{aligned} y &= \frac{202000 - 98000}{1990 - 1970}(t - 1970) + 98000 \\ &= 5200(t - 1970) + 98000 \end{aligned}$$

So, $j(t) = 5200(t - 1970) + 98000$. The graph is sketched in part (b) along with the two data points.

(d) (5pt) When will the value of a house in La Jolla exceed the value of a house in Seattle by exactly \$5,000? (Carry out to one decimal place.)

We need to write down the equation to solve; this amounts to translating the words into an equation:

$$\begin{aligned} \text{value house in La Jolla} &\quad \textit{exceed} \quad \text{the value of house Seattl eby exactly \$5,000} \\ j(t) &= s(t) + 5000 \\ 5200(t - 1970) + 98000 &= 7000(t - 1970) + 40000 + 5000 \\ 5200t - 10146000 &= 7000t - 13745000 \\ 3599000 &= 1800t \\ 1999.4 &= t \end{aligned}$$

The answer is the year 1999.4 (rounded to one decimal place.)

(e) (2pt) Consider the function $s(t)$ on the NEW domain $1990 \leq t \leq 2020$. What is the range of this function?

The graph of this function is a piece of the line graphed in (b) for the graph of $s(t)$. You need to consider only the part of the line which is above the domain on the $t$-axis. That means the part of the graph above the interval $1990 \leq t \leq 2020$. On this interval, you can see from the picture that $s(1990) = 180000$ is the smallest value and $s(2020) = 390000$ is the largest value. Conclude the range is $180000 \leq y \leq 390000$.

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2. (2pt) Consider the function

$$f(x) = \frac{-x}{x^2 + 1}$$

on the domain of all real numbers. Compute

(a) $f(-3) = \frac{-(-3)}{(-3)^2+1} = \frac{3}{10} = 0.3$.

(b) $f(1 - 2x) = \frac{-(1-2x)}{(1-2x)^2+1} = \frac{2x-1}{(1-2x)^2+1} = \frac{2x-1}{4x^2-4x+2}$