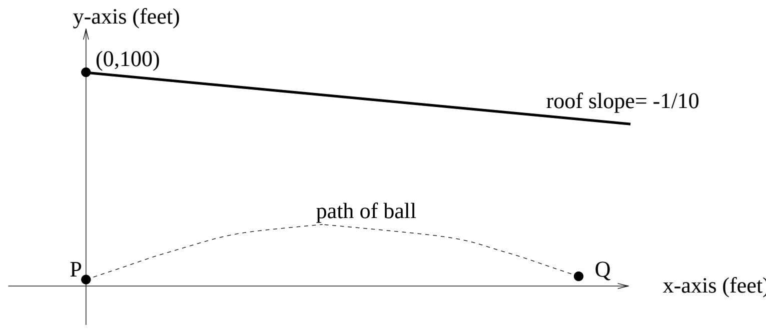
Math 120A
January 22, 1998
Quiz #3 (20 points)

Name ______

TA: ______

Instructions: You have 40 minutes. You **MUST** show work for credit. If in doubt, ask for clarification.

1. (15 pts) A baseball is hit inside a building with a sloped roof as pictured. You impose coordinates as pictured, so that the baseball is hit (at the point P) when it is directly above the origin. Assume the flight path of the baseball is the graph of the function $f(x) = -0.003x^2 + 0.6x + 3$.



(a) (1pt) Find the height of the ball above the ground when it was hit.(Round to nearest foot.)

Plug in $x = 0$ into function: $f(0) = 3$ feet.

(b) (3pt) A player catches the ball when it is 6 feet above the ground. Find the coordinates of the ball when it is caught; this is at the point Q in the picture. (Round to nearest foot.)

Need to solve $6 = f(x)$ for x; will require quadratic formula:

$$6 = -0.003x^2 + 0.6x + 3$$
$$0 = -0.003x^2 + 0.6x - 3$$

$$x = \frac{-0.6 \pm \sqrt{0.6^2 - 4(-3)(-0.003)}}{2(-0.003)} = \frac{-0.6 \pm 0.5692}{-0.006} = 5.13, \quad 194.87.$$

Conclude $Q = (195, 6)$, to nearest foot.

(c) (3pt) Put the function $f(x)$ into vertex form and find the vertex of the flight path of the ball. (Round to nearest foot).

Follow formula from text (class) to put into vertex form, using fact $a = -0.003, b = 0.6$:
$f(x) = a(x-h)^2 + k = -0.003(x-h)^2 + k$
$a = -0.003$
$h = \frac{-b}{2a} = \frac{-0.6}{-0.006} = 100,$
$k = f(100) = -0.003(100^2) + 0.6(100) + 3 = 33.$
So $f(x) = -0.003(x-100)^2 + 33$ and the vertex is (100,33).

(d) (2pt) Find a function $g(x)$ whose graph will model the roof of the building.

Use slope intercept formula: $g(x) = -0.1x + 100$.

(e) (2pt) Find the vertical distance between the roof and the ball the instant it is caught (at the point Q). (Round to nearest foot.)

The x coord of Q is 195 by part (b). Need to find:
$g(195) - f(195) = (-0.1(195) + 100) - (-0.003(195^2) + 0.6(195) + 3) = 74.57$.
Conclude distance is 75 feet, to the nearest foot.

(f) (4pt) Find the smallest vertical distance between the ceiling and the baseball. (Round to nearest foot.)

We need a function $v(x)$ that calculates the vertical height between the baseball and the roof:
$v(x) = (\text{height of roof at } x)$ - (height of ball at x)
$= g(x) - f(x) = -0.1x + 100 - (-0.003x^2 + 0.6x + 3) = 0.003x^2 - 0.7x + 97$.
Need to put this in vertex form, where $a = 0.003, b = -0.7$:
$v(x) = a(x - h)^2 + k$

$a = 0.003$,

$h = \frac{-b}{2a} = \frac{0.7}{0.006} = 116.67$.

$k = v(h) = 0.003(116.67)^2 - 0.7(116.67) + 97 = 56.2$

Conclude $v(x) = 0.003(x - 116.67)^2 + 56.2$. This is a quadratic function with positive leading coefficient, so the graph vertex corresponds to a minimum value. That means the SMALLEST value of $v(x)$ is 56.2; i.e. the smallest vertical distance to the roof is 56 feet, rounded to the nearest foot.

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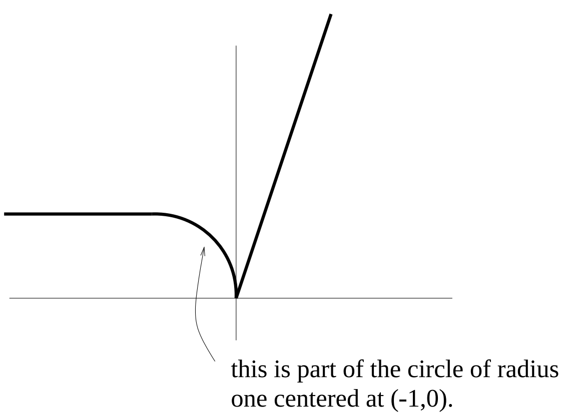
2. (5pt) Sketch the graph of this multi-part function below. Solve the equation $m(x) = 1/2$.

$$m(x) = \begin{cases} 1 & \text{if } x \le -1 \\ \sqrt{1 - (x+1)^2} & \text{if } -1 \le x \le 0 \\ 3x & \text{if } x \ge 0 \end{cases}$$



To solve the equation $m(x) = 1/2$, we consider each case of the multipart rule individually. In the first case, the equation becomes $1 = 1/2$, which is false; there is no solution. In the second case, the equation is $\sqrt{1 - (x+1)^2} = 1/2$. This implies $1 - (x - 1)^2 = 1/4$, so $(x - 1)^2 = 3/4$; this equation has two solutions: -0.134 or -1.866. Only -0.134 is allowed in the domain. Finally, in the third case, we get $3x = 1/2$, so $x = 1/6$. Conclude there are two solutions: $x = -0.134, 1/6$.