1. Recall that $f(x) = \dfrac{3+x}{x}$. We wish to simplify $\dfrac{f(x+h)-f(x)}{h}$ as much as possible, so we first need to figure out what $f(x+h)$ is. Since $f(x) = \dfrac{3+x}{x}$, we get $f(x) = \dfrac{3+x+h}{x+h}$. Now simplifying, we get

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{1}{h}\left[\frac{3+x+h}{x+h} - \frac{3+x}{x}\right] \\
&= \frac{1}{h}\left[\frac{(3+x+h)x}{x(x+h)} - \frac{(3+x)(x+h)}{x(x+h)}\right] \\
&= \frac{1}{h}\cdot\frac{3x+x^2+hx-(3x+3h+x^2+hx)}{x(x+h)} \\
&= \frac{-3h}{hx(x+h)} \\
&= \frac{-3}{x(x+h)}.
\end{aligned}$$

This is as simplified as possible.

2. We wish to find the linear velocity of wheel C. The key formula in this case is $v = r\omega$, where for a given wheel, v is the linear velocity, r is the radius, and ω is the angular velocity (in *radians* per unit time). We will use subscripts to distinguish between wheels, so, for example, we wish to find v_C and we are told, for example, that $\omega_A = 200$ RPM and $r_B = 10$ cm.

Since wheels A and B are connected at the hub, they rotate at the same angular speed, so $\omega_A = \omega_B$. We may thus find wheel B's linear speed v_B, as we know r_B and ω_B. Since wheels B and C are connected by a belt, their linear velocities are the same, so $v_C = v_B$. This is what we wish to find.

Now to the calculations. We first convert ω_B to radians per second:

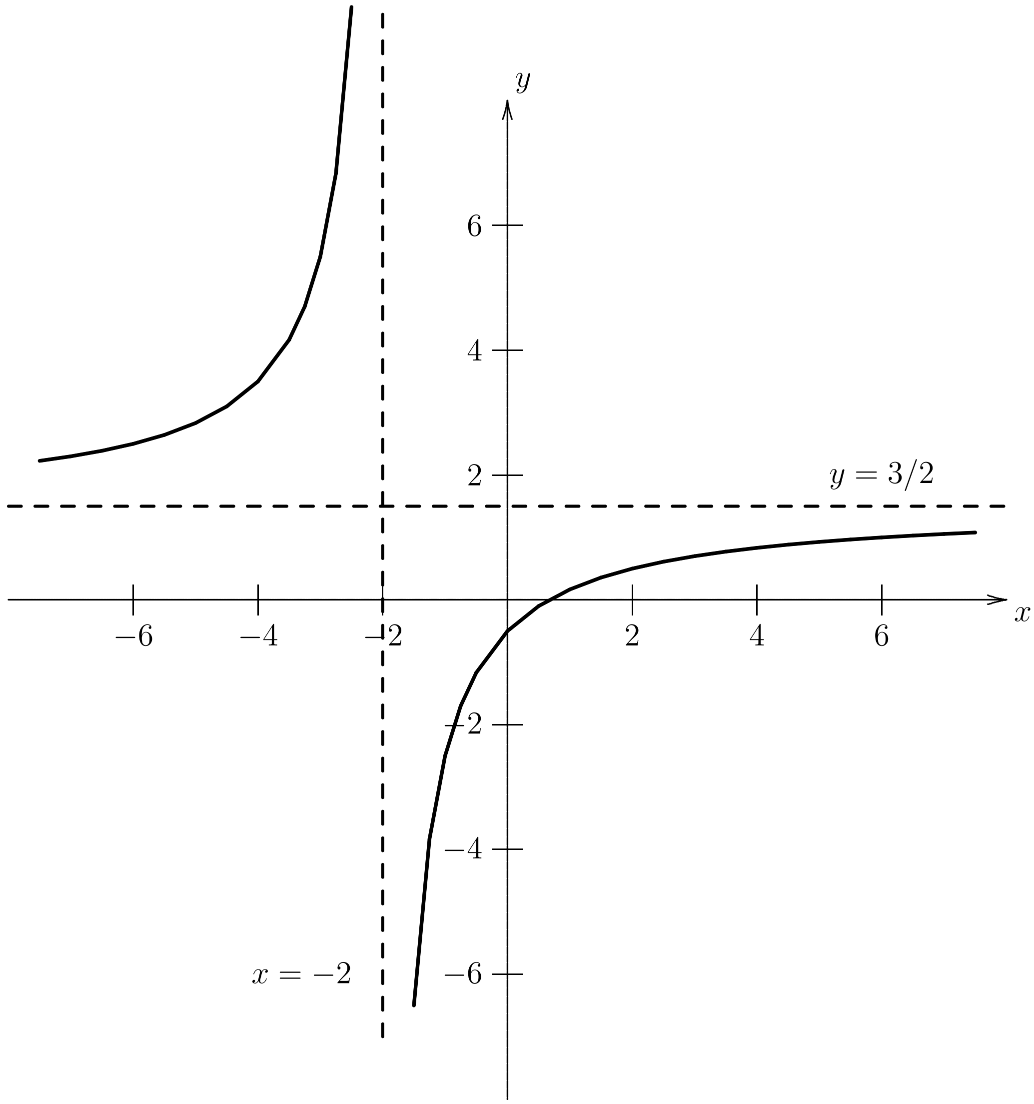
$$\omega_B = \omega_A = \left(200\ \frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi\ \text{rads}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{secs}}\right) = \frac{20\pi}{3}\ \frac{\text{rads}}{\text{sec}}.$$

Next, we find wheel B's linear velocity using the formula $v = r\omega$:

$$v_B = r_B\omega_B = (10\ \text{cm})\left(\frac{20\pi\ \text{rads}}{3\ \text{sec}}\right) = \frac{200\pi\ \text{cm}}{3\ \text{sec}} \approx 209.44 \text{ cm per second.}$$

This v_B, which is equal to v_C since wheels B and C are connected by a belt. Thus the linear velocity of wheel C is $\dfrac{200\pi}{3}$ cm per second, or roughly 209.44 cm per second.

3. (a) Recall that $f(x) = \dfrac{3x-2}{2x+4}$. The y-intercept is where $x = 0$, this occurs at the point $(0, f(0)) = (0, -1/2)$. The zeroes of this function are where $f(x) = 0$; this occurs only at $(2/3, 0)$. The vertical asymptote is when the denominator $2x+4$ is zero; it is the line $x = -2$. The horizontal asymptote is found by writing $f(x) = \frac{3-\frac{2}{x}}{2+\frac{4}{x}}$, so as x gets large $f(x)$ tends to $3/2$. Thus the horizontal asymptote is $y = 3/2$. The graph of this function looks like this:



(b) The domain of $f(x)$ is the values on which $f(x)$ is defined, namely $\{x \ : \ x \neq -2$ (all x except $x = -2$). Similarly, the range of $f(x)$ is all possible values of y on the graph of $y = f(x)$. From our graph, this is clearly all values except $y = 3/2$: $\{y \ : \ y \neq 3/2\}$.

(c) The value $f^{-1}(10)$ is the value of x for which $f(x) = 10$. That is, we must solve $\dfrac{3x-2}{2x+4} = 10$. We multiply through by $2x+4$ and get $3x - 2 = 10(2x+4) = 20x + 40$, which simplifies to $17x = -42$, or $x = -42/17$.

Alternatively, we could find the function $f^{-1}(x)$ and plug in $x = 10$. We do this by solving for x in $y = \dfrac{3x-2}{2x+4}$. We get $3x - 2 = y(2x+4) = 2xy + 4y$. After simplifying, this is $3x - 2yx = 4y + 2$ or $x = \dfrac{4y+2}{-2y+3}$. Thus $f^{-1}(y) = \dfrac{4y+2}{-2y+3}$ or (equivalently) $f^{-1}(x) = \dfrac{4x+2}{-2x+3}$. Plugging in $x = 10$, we have $f^{-1}(10) = \dfrac{42}{-17} = -42/17$, as above.

4. (a) Jody's angular speed is given: it's $\omega = 3/4$ RPM. We convert this to radians per second:

$$\omega = \left(\frac{3}{4}\ \frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi\ \text{rads}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{secs}}\right) = \frac{\pi\ \text{rads}}{40\ \text{sec}}.$$

Thus Jody's angular speed is $\omega = \pi/40$ radians per second, or roughly 0.785 radians per second. (Note: since Jody is traveling clockwise, perhaps the more correct answer is that $\omega = -\pi/40$ radians per second. This negative sign will become important in part (c), below.)

(b) We will use the formula $\theta = \omega t$ to find the angle that Jody has passed through, and then $s = r\theta$ to find the length of the arc that she travels. We've found ω in part (a), so we get $\theta = \left(\frac{\pi}{40}\ \frac{\text{rads}}{\text{sec}}\right)(70\ \text{secs}) = 7\pi/4$ radians. Since the radius of the wheel is 45 feet, Jody travels a distance of $s = (45\ \text{feet})(7\pi/4\ \text{radians}) = 78.75\pi$ feet, or roughly 247.40 feet.

(c) Jody's coordinates are given by the parametric equations

$$x(t) = x_c + r\cos(\omega t + \theta_0)$$
$$y(t) = y_c + r\sin(\omega t + \theta_0),$$

where $(x_c, y_c) = (0, 50)$ are the coordinates of the center of the circle, $r = 45$ is the radius, $\omega = -\pi/40$ radians per second is the angular velocity, and θ_0 is the initial angle (that is, Jody's "standard" angle at $t = 0$). All that's left for us is to find θ_0.

The angle θ_0 is the angle that the point J makes with the horizontal ray going to the right from the center of the circle. This angle is $\pi/2$ more than the angle that Jody passes through (that we found in part (b), above). Thus one possible θ_0 is $\frac{7\pi}{4} + \frac{\pi}{2} = \frac{9\pi}{4}$; another is 2π less than this one, or $\pi/4$. Thus Jody's coordinates at time t are given by

$$x(t) = 45\cos\left(-\frac{\pi}{40}t + \frac{\pi}{4}\right)$$
$$y(t) = 50 + 45\sin\left(-\frac{\pi}{40}t + \frac{\pi}{4}\right).$$

5. (a) The coordinates $(x(t), y(t))$ of plane A at time t minutes are of the linear parametric equations, so they are of the form

$$x(t) = x_0 + v_x t$$
$$y(t) = y_0 + v_y t,$$

where $(x_0, y_0) = (1, 12)$ is the position of plane A at time $t = 0$, v_x is the horizontal velocity of plane A, and v_y is the vertical velocity of plane A. (Note that since t is in minutes and the coordinate system is in miles, we want v_x and v_y to be in units of miles per minute.)

To compute the velocities, we need to know when plane A is at two different points. We know that at $t = 0$ the plane is at $(1, 12)$, but we don't know t when the plane is at $(10, 0)$. We do know that the plane is traveling 150 miles per hour, so we can find the time by finding the distance: $d = \sqrt{(1-10)^2 + (12-0)^2} = 15$ miles. Since distance equals speed times time, the time is $t = \frac{d}{v} = \frac{15\ \text{miles}}{150\ \text{mph}} = 0.1$ hours, or 6 minutes. From this we get the horizontal and vertical velocities:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{10 - 1\ \text{miles}}{6 - 0\ \text{mins}} = 1.5\ \text{miles per minute}$$

and

$$v_y = \frac{\Delta y}{\Delta t} = \frac{0 - 12\ \text{miles}}{6 - 0\ \text{mins}} = -2\ \text{miles per minute}.$$

Thus plane A's coordinates at time t minutes are

$$x(t) = 1 + 1.5t$$
$$y(t) = 12 - 2t.$$

(b) We want to find the equation of the line $y = mx + b$ of travel for plane B. We are told, through the parametric equations, that the initial position (at $t = 0$) of plane B is $(x, y) = (-6, 0)$, the horizontal velocity of the plane is $v_x = 4.5$ miles per minute, and the vertical velocity is $v_y = 0.5$ miles per minute. The slope of the line of travel is $m = v_y/v_x = 0.5/4.5 = 1/9$, so the equation is $y = \frac{1}{9}x + b$. By plugging in $(x, y) = (-6, 0)$, we find that the equation is $y = \dfrac{1}{9}x + \dfrac{6}{9}$ or $y = \dfrac{1}{9}x + \dfrac{2}{3}$.

(c) To find when the two planes are 5 miles apart, we set the distance between them equal to 5 miles and solve for t. The distance between the two planes is

$$\begin{aligned} d &= \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \\ &= \sqrt{((1 + 1.5t) - (-6 + 4.5t))^2 + ((12 - 2t) - 0.5t)^2} \\ &= \sqrt{(7 - 3t)^2 + (12 - 2.5t)^2} \\ &= \sqrt{15.25t^2 - 102t + 193}. \end{aligned}$$

We set this equal to 5 miles and square both sides:

$$25 = 15.25t^2 - 102t + 193,$$

or

$$15.25t^2 - 102t + 168 = 0.$$

Using the quadratic formula, we get two times when the planes are 5 miles apart:

$$t = \frac{102 \pm \sqrt{156}}{30.50} = \frac{204 \pm 4\sqrt{39}}{61} \approx 2.93 \text{ or } 3.75 \text{ minutes.}$$

6. Recall that the temperature T (in degrees Celsius) of the coffee and time t (in minutes since it was poured) are related by the formula

$$t = -25 \ln\left(\frac{T - 20}{75}\right).$$

(a) The coffee 40° C when $T = 40$. This occurs at time

$$t = -25 \ln\left(\frac{40 - 20}{75}\right) = -25 \ln\left(\frac{4}{15}\right) \approx 33.04 \text{ minutes}$$

since the coffee was poured.

(b) For this question, we're looking for T when $t = 0$. That is, we wish to solve the equation

$$0 = -25 \ln\left(\frac{T - 20}{75}\right)$$

for the temperature T. If we divide both sides by -25, then exponentiate, we get $(T - 20)/75 = e^0 = 1$, or $T = 95$ Celsius.

(c) For this question, we're looking for T when $t = 10$. That is, we wish to solve the equation

$$10 = -25 \ln\left(\frac{T - 20}{75}\right)$$

for the temperature T. If we divide both sides by -25, then exponentiate, we get $(T - 20)/75 = e^{-10/25}$, or $T = 20 + 75e^{-2/5} \approx 70.27$ Celsius.

7. (a) We wish to write the depth of the water as a sinusoidal function $d(t) = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$, where $d(t)$ is in feet and the time t is the number of hours since midnight. We are told that the maximum depth is 25 feet, and the minimum depth is 5 feet. Thus the mean is $D = (25 + 5)/2 = 15$ and the amplitude is $A = 25 - 15 = 10$ (or $15 - 5 = 10$, or $(25 - 5)/2 = 10$). Also, we know that it takes 3.5 hours to go from a maximum (high tide) to a minimum (low tide). This is also half of a period, so $3.5 = B/2$, or $B = 7$ hours. Finally, to find the phase shift C, we use the fact that at $t = 6.5$ (6:30 AM) the water level is at the mean and *falling*. This is exactly half a period (or 3.5 hours) away from when the water level is at the mean and *rising* (which is a possible value of C). Thus possible values of C are $6.5 - 3.5 = 3$, or $6.5 + 3.5 = 10$. Putting this together, we have the sinusoidal function is $d(t) = 10\sin\left(\frac{2\pi}{7}(t - 3)\right) + 15$.

(b) This question asks for the depth of the water at 1:00 AM, or $t = 1$. This is simply $d(1) = 10\sin\left(\frac{2\pi}{7}(1 - 2)\right) + 15 \approx 5.251$ feet.

(c) We wish to find the times at which $d(t) = 10$, so that we can say when the depth of the water is at least 10 feet. If we solve $d(t) = 10\sin\left(\frac{2\pi}{7}(t - 3)\right) + 15 = 10$, we get $\sin\left(\frac{2\pi}{7}(t - 3)\right) = -1/2$. Thus we want to find possible solutions to $\sin(\theta) = -1/2$. Two solutions to this are $\theta_1 = \sin^{-1}(-1/2) = -\pi/6 \approx -0.523599$ and $\theta_2 = \pi - \sin^{-1}(-8/10) \approx 3.665191$. Rewriting these in terms of times, we have $\frac{2\pi}{7}(t_1 - 3) = \sin^{-1}(-1/2) = -\pi/6$ and $\frac{2\pi}{7}(t_2 - 3) = \pi - \sin^{-1}(-8/10) = 7\pi/6$. Solving, these are $t_1 = 3 + \frac{7}{2\pi} \cdot \left(-\frac{\pi}{6}\right) = 3 - \frac{7}{12} = 29/12$ (the principal solution) and $t_2 = 3 + \frac{7}{2\pi} \cdot \frac{7\pi}{6} = 3 + \frac{49}{12} = 85/12$ (the symmetry solution).

By adding and subtracting multiples of the period (7 hours), we can find *all* times when the depth is precisely 10 feet. During the first 24 hours (from $t = 0$ to $t = 24$) these times are $t = 85/12 - 7 = 1/12$, $t = 29/12$, $t = 85/12$, $t = 29/12 + 7 \approx 9.42$, $t = 85/12 + 7 \approx 14.08$, $t = 29/12 + 14 \approx 16.42$, $t = 85/12 + 14 \approx 21.08$, and $t = 29/12 + 21 \approx 24.58$. (The next time is $t = 85/12 + 21 \approx 28.08$.)

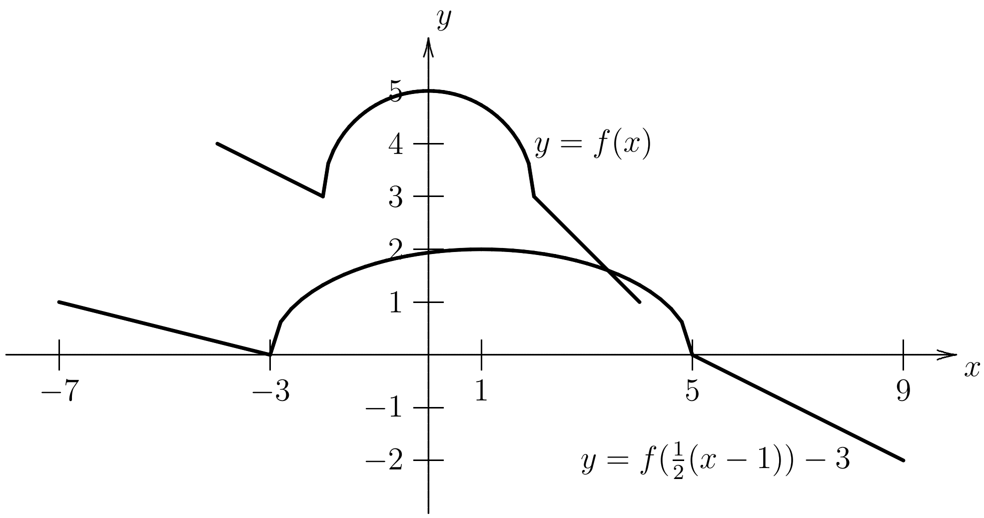
Looking at the graph, or noticing that the water is 15 feet deep at $t = 6.5$, we see that the water is at least 10 feet deep from $t = 0$ to $t = 1/12$, from $t = 29/12$ to $t = 85/12$ (and $t = 6.5$ sits in this interval), from $t = 29/12 + 7$ to $t = 85/12 + 7$, from $t = 29/12 + 14$ to $t = 85/12 + 14$, and from $t = 29/12 + 21$ to $t = 24$. This is a total time of $\frac{1}{12} + \frac{56}{12} + \frac{56}{12} + \frac{56}{12} + \frac{7}{12} = \frac{176}{12} \approx 14.67$ hours.

8. (a) The formula for the value of Linda's house in year $1996 + x$ is an exponential function, so it is of the form $H(x) = H_0 \cdot b^x$. We're told that $H(0) = 150,000$ and $H(5) = 210,000$. Plugging these in, we find $H_0 = 150,000$ and $210,000 = 150,000 \cdot b^5$. This means $b^5 = 1.4$, or $b = (1.4)^{1/5} = \sqrt[5]{1.4} \approx 1.069610376$. The final formula is thus $H(x) = 150,000(1.4)^{x/5} \approx 150,000(1.069610376)^x$.

(b) The value of Linda's home in 2005 is $H(9) = 150,000(1.4)^{9/5} \approx \$274,866.44$.

(c) We begin by finding x so that $H(x) = 2(150,000)$, or $(1.4)^{x/5} = 2$. We take the natural logarithm of this equation to get $\frac{x}{5}\ln(1.4) = \ln(2)$, or $x = \frac{5\ln(2)}{\ln(1.4)} \approx 10.30$ years **after 1996**.

9. (a) and (b) The graphs for the first two parts look like these:



(c) The function $y = \sqrt{f(\frac{1}{2}(x-1)) - 3}$ is defined on the domain $\{x \ : \ -7 \leq x \leq 5\}$. This is the largest possible domain, as is clear from the graph (since $f(\frac{1}{2}(x-1)) - 3 < 0$ or undefined outside this set).