1 (a) Since the hill is a line that rises vertically 20 feet every 100 horizontal feet, the slope of this line is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{20}{100} = 2/10$. The y-interecept of this line is $b = 0$, as the line passes through the origin of our coordinate system. Thus the equation is $y = 2x/10$ or $y = 0.2x$.

(b) The cliff, off which the ball is hit, is the same height as the ball at $t = 0$. Thus the height is $y(0) = -\dfrac{1}{50} \cdot 0^2 + \dfrac{95}{10} \cdot 0 + 250 = 250$ feet.

(c) The coordinates of the highest point **over the hill** that the ball reaches can be found by looking at the height of the ball over the hill. This is given by $h(x)$:

$$\begin{aligned} h(x) &= y_{\text{ball}} - y_{\text{hill}} \\ &= \left(-\frac{1}{50}x^2 + \frac{95}{10}x + 250\right) - \left(\frac{2}{10}x\right) \\ &= -\frac{1}{50}x^2 + \frac{93}{10}x + 250. \end{aligned}$$

The height $h(x)$ is greatest at the vertex of $h(x)$:

$$x = -\frac{b}{2a} = -\frac{93/10}{2(-1/50)} = \frac{465}{2} = 232.5.$$

Plugging this in to the path of the ball (not into $h(x)$), we see that at it's greatest height over the hill, the ball is at the point $(x, y) = (232.5, 1377.625)$.

(d) The ball lands on the hill where the parabola and the line intersect; that is, where

$$-\frac{1}{50}x^2 + \frac{95}{10}x + 250 = \frac{2}{10}x,$$

or

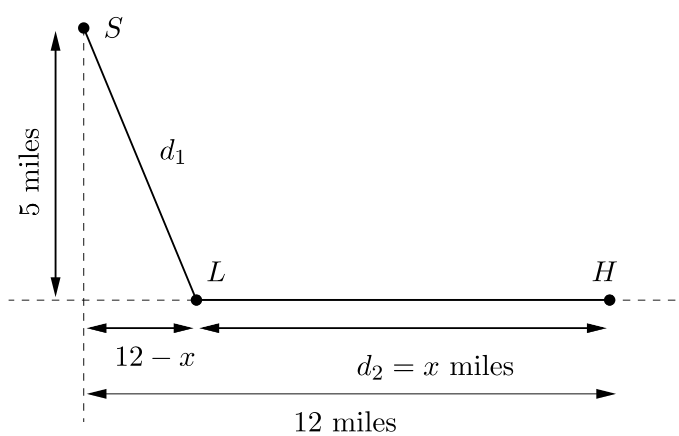
$$-\frac{1}{50}x^2 + \frac{93}{10}x + 250 = 0.$$

Plugging in to the quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9.3 \pm \sqrt{9.3^2 - 4(-1/50)250}}{2(-1/50)} \approx 490.485 \text{ and } -25.485.$$

The x coordinate of the point where the ball lands on the hill is the positive solution: $x \approx 490.485$.

2 I have re-drawn the picture so that it includes several useful labels.



First, d_1 is the distance traveled at sea, and $d_2 = x$ is the distance traveled on land. We use the Pythagorean theorem to find d_1: $d_1^2 = 5^2 + (12-x)^2$. Thus $d_2 = \sqrt{25+(12-x)^2}$.

Let t_1 and t_2 be the times that it takes to cover distances d_1 and d_2. Since the speed at sea is 3 mph, we have $d_1 = (3 \text{ mph})t_1$, or $t_1 = d_1/3$. Similarly, the speed on land is 5 mph, so $d_2 = (5 \text{ mph})t_2$, or $t_2 = d_2/5$. Plugging in our formulas for $d_1 = \sqrt{25+(12-x)^2}$ and $d_2 = x$, we get $T(x) = \dfrac{\sqrt{25+(12-x)^2}}{3} + \dfrac{x}{5}$ hours.

3 (a) The line passes through the points $(30,0)$ and $(0,20)$. Thus the slope of this line is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{0-20}{30-0} = -2/3$. The y-intercept is $b = 20$ (from the picture or plugging in either point); we therefore have $y = -\dfrac{2}{3}x + 20$.

(b) The circle has a diameter from the origin $(0,0)$ to the point $(30,0)$. Thus the center is at $(15,0)$ and the radius is 15. Hence the circle has equation $(x-15)^2 + (y-0)^2 = 15^2$, or (if you prefer) $x^2 - 30x + y^2 = 0$.

(c) The point P is one of the points of intersection between the line and the circle (the other is $(x,y) = (30,0)$). We find this point by replacing y in the circle equation by the y in the equation for the line:

$$(x-15)^2 + (-\frac{2}{3}x + 20 - 0)^2 = 15^2.$$

This simplifies to

$$x^2 - 30x + 225 + \frac{4}{9}x^2 - \frac{80}{3}x + 400 = 225,$$

or

$$\frac{13}{9}x^2 - \frac{170}{3}x + 400 = 0.$$

We can multiply the entire equation by 9 to make it simpler:

$$13x^2 - 510x + 3600 = 0.$$

We can solve this either using the quadratic formula:

$$x = \frac{+510 \pm \sqrt{(-510)^2 - 4(13)(3600)}}{2(13)} = \frac{510 \pm \sqrt{72900}}{26} = \frac{510 \pm 270}{26} = \frac{120}{13} \text{ or } 30.$$

We could have also factored: $13x^2 - 510x + 3600 = (13x-120)(x-30) = 0$, so $x = 120/13$ or 30.

Since $x = 120/13$, $y = -\frac{2}{3} \cdot \frac{120}{13} + 20 = \frac{280}{13}$, so P has coordinates $(x,y) = (120/13, 280/13)$.

4 (a) On the domain $-6 \le x \le 0$, the function $f(x)$ is a line, through the points $(-6,0)$ and $(0,3)$. Thus the equation for this line is $y = 0.5x + 3$. For $x \ge 0$, the function $f(x)$ is a parabola with vertex $(3,6)$ passing through the point $(0,3)$. Thus the equation of this parabola is $y = a(x-3)^2 + 6$ for some a. We determine a by plugging in the point $(x,y) = (0,3)$: $3 = a(6-3)^2 + 6$, or $a = -1/3$. Hence $f(x) = -\frac{1}{3}(x-3)^2 + 6$ for $x \ge 0$.

The final part of the problem is to determine the end of the domain: where does our parabola hit the x-axis? This happens when $y = 0$, so we solve: $0 = -\frac{1}{3}(x-3)^2 + 6$, or $x = 3 \pm 3\sqrt{2}$. Since $x \geq 0$, our domain for the parabolic part of $f(x)$ is $0 \leq x \leq 3 + 3\sqrt{2} \approx 7.24$. Thus the function is

$$f(x) = \begin{cases} \frac{1}{2}x + 3 & \text{if } -6 \leq x < 0 \\ -\frac{1}{3}(x-3)^2 + 6 & \text{if } 0 \leq x \leq 3 + 3\sqrt{2} \end{cases}$$

(b) Here is the graph of $y = 2f(3x)$ (the solid line), with a graph of $y = f(x)$ (the dashed line) included for reference: