MATH 120B **EXAM TWO SOLUTIONS** WINTER, 2002

$\boxed{1}$ (a) It's simpler to describe the solution if you've done part (b) first. So go read the solution to (b) and come back. Okay?

The point a is on the mean line ($y = D = 6$) one quarter of a period after the first maximum (which is at $t = 0.75$ seconds). Thus the time for point a is $t = 0.75 + (1/4) * B = 0.75 + 2/4 = 1.25$ seconds, so the coordinates for a are $(1.25, 6)$.

The point b is a maximum (so $y = 10$ cm), which happens one period (2 seocnds) after the first maximum (at $t = 0.75$). Thus the coordinates for b are $(2.75, 10)$.

Finally, the point c is at a minimum (so $y = 2$ cm), half a period (or 1 second) after the point b. Thus c is $(3.75, 2)$.

(b) The largest (maximum) value is 10 cm, the smallest (minimum) value is 2 cm, so the mean is $D = (10 + 2)/2 = 6$ cm. The amplitude is therefore $A = \max - D = 10 - 6 = 4$ (or $A = D - \min = 6 - 2 = 4$) cm. The period is the time between consecutive maxima, so $B = 2$ seconds. Finally, we're told that the time at the maximum is $t_{\max} = 0.75$ seconds, so one possible phase shift is $C = t_{\max} - B/4 = 0.25$ seconds. Thus our formula is

$$d(t) = 4\sin\left(\frac{2\pi}{2}(t - 0.25)\right) + 6.$$

(c) Now we are asked to find the first time after $t = 1$ with $d(t) = 8$ cm. Thus,

$$4\sin\left(\frac{2\pi}{2}(t - 0.25)\right) + 6 = 8,$$

or

$$\sin\left(\frac{2\pi}{2}(t - 0.25)\right) = 0.5.$$

We've reduced the problem to solving $\sin(\theta) = 0.5$ for θ, then setting $\theta = \frac{2\pi}{2}(t - 0.25)$ and solving for t.

First, let's find what solutions there are for $\sin(\theta) = 0.25$. The first is the principal solution, $\theta_1 = \sin^{-1}(0.5) = \frac{\pi}{6}$ radians. The second, the symmetry solution, is $\theta_2 = \pi - \sin^{-1}(0.5) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ radians. These correspond to $t_1 = 0.25 + \frac{2}{2\pi}\sin^{-1}(0.5) = \frac{5}{12}$ minutes and $t_2 = 0.25 + \frac{2}{2\pi}(\pi - \sin^{-1}(0.5)) = \frac{13}{12}$ minutes. All other solutions follow from these two by adding multiples of the period (which is $B = 2$ here) to t_1 or t_2. Thus the solutions in the first two seconds are just $t_1 = 5/12$ and $t_2 = 13/12$, which mean that the mass spends $13/12 - 5/12 = 2/3$ seconds at least 8 cm from the wall during this period.

$\boxed{2}$ (a) We wish to solve for x so that $\sin^{-1}\left(\sqrt{x^2 - 1}\right) = \frac{\pi}{4}$. We take the sine of both sides and get $\sqrt{x^2 - 1} = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$. Squaring both sides and solving, we get $x^2 = 3/2$, or $x = \pm\sqrt{3/2}$. (Either answer is acceptable.)

(b) This is almost the same as part (a); here we solve $y = \sin^{-1}\left(\sqrt{x^2 - 1}\right)$ for x. (We'll then have $f^{-1}(y)$, so we'll then switch y for x. Some people prefer switching x and y at the outset.) We solve by taking sine of both sides to get $\sqrt{x^2 - 1} = \sin(y)$, so then $x = \pm\sqrt{1 + \sin^2(y)}$. Thus our inverse function is either $f^{-1}(x) = +\sqrt{1 + \sin^2(x)}$ or $f^{-1}(x) = -\sqrt{1 + \sin^2(x)}$ (but not both).

$\boxed{3}$ Both parts of this problem are, like all belt-wheel problems, simply applications of the formula $v = r\omega$. We'll use subscripts to distinguish between the various wheels (for example, r_A for the radius of wheel A, ω_B for the angular speed of wheel B, and v_C for the linear velocity of wheel C).

(a) Here we're told $r_A = 25$ cm and we're asked for v_A. We aren't given ω_A directly, but we're told that $\omega_B = 90$ RPM. Moreover, we're told that wheels A and B are joined at the axle, so that they rotate together, which means that $\omega_A = \omega_B = 90$ RPM. Thus we can use $v = r\omega$ to find v for wheel A. Recall that this formula requires ω to be in *radians* per unit of time (and also that we're asked for units of cm per second), so we convert in the process:

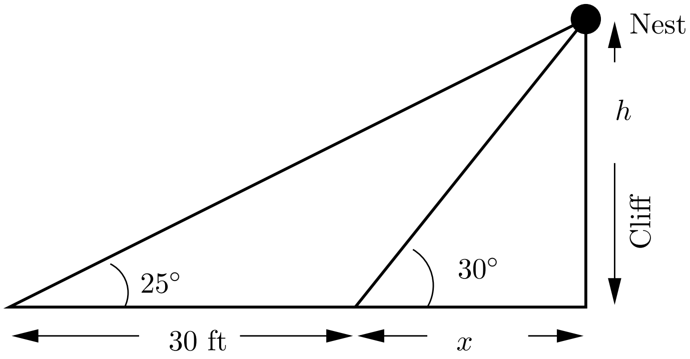
$$v_A = r_A\omega_A = (25 \text{ cm})\left(90\,\frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi \text{ rads}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 75\pi \text{ cm/sec} \approx 235.619 \text{ cm/sec}.$$

(b) We relate wheels B and C by noticing that the belt connecting them implies $v_B = v_C$, or $r_B\omega_B = r_C\omega_C$ (when both angular speeds ω_B and ω_C are written in radians per time). That is, we get

$$(10 \text{ cm})\left(90\,\frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi \text{ rads}}{1 \text{ rev}}\right) = r_C\left(40\,\frac{\text{revs}}{\text{min}}\right)\left(\frac{2\pi \text{ rads}}{1 \text{ rev}}\right),$$

so $r_C = 22.5$ cm.

$\boxed{4}$ To the right I've redrawn the picture, with the labels x and h added. The key to this problem is to use the tangents of the two angles:



$$\tan(30^\circ) = \frac{h}{x}$$

and

$$\tan(25^\circ) = \frac{h}{x+30}.$$

(a) In this part we're asked to find what I've labeled as x. From the two tangent equations (see above), we have

$$h = x\tan(30^\circ) \qquad \text{and} \qquad h = (x+30)\tan(25^\circ).$$

Thus $x\tan(30^\circ) = (x+30)\tan(25^\circ)$ or, solving, $x = \dfrac{30\tan(25^\circ)}{\tan(30^\circ) - \tan(25^\circ)} \approx 125.981$ feet.

(b) Now we're asked to find h. We've already found x and seen that $h = x\tan(30^\circ)$, so $h = \dfrac{30\tan(25^\circ)\tan(30^\circ)}{\tan(30^\circ) - \tan(25^\circ)} \approx 72.735$ feet.