Math 120, Winter 2003 **Final Examination - Hints and Answers**

1. (a) ANSWER: $\dfrac{f(x+h)-f(x)}{h} = 2x+h$

   (b) ANSWER: There are no zeros. The line $y = \frac{1}{4}$ is the horizontal asymptote. The lines $x = -5$ and $x = 4$ are the vertical asymptotes.

   (c) $g^{-1}(x) = \dfrac{1}{e^x - 1}$

   (d) HINT: Start by taking the ln of both sides:

   $$\ln 7^{(a^2-5a)} = \ln 3 \Rightarrow (a^2-5a)\ln 7 = \ln 3 \Rightarrow a^2 - 5a = \frac{\ln 3}{\ln 7} \Rightarrow a^2 - 5a - \frac{\ln 3}{\ln 7} = 0.$$

   Use the quadratic formula to solve for a.
   ANSWER: $a = 5.1105$ or -0.1105

2. (a) ANSWER: $D_f = [-6, 5]$, $R_f = [-1, 2]$

   (b) ANSWER:

   $$f(x) = \begin{cases} -1 & \text{if } -6 \le x \le -3 \\ -1 + \sqrt{4-(x+1)^2} & \text{if } -3 \le x \le 1 \\ \frac{3}{4}x - \frac{7}{4} & \text{if } 1 \le x \le 5 \end{cases}$$

   (c) ANSWER: The y-intercept is $\sqrt{3} - 1$. The x-intercepts are $x = -1 \pm \sqrt{3}$ and $x = \frac{7}{3}$.

   (d) HINTS: To find the domain, notice that x is in the domain of $g(x)$ if, and only if, $\frac{1}{2}(x-1)$ is in the domain of $f(x)$. So, solve the inequality $-6 \le \frac{1}{2}(x-1) \le 5$ for x. The range of $g(x)$ will be the same as the range of $f(x)$.
   ANSWER: $D_g = [-11, 11]$, $R_g = [-1, 2]$

3. (a) HINT: You'll earn \$900 for selling 60 CDs ($15\times\$60$). You'll spend $C(60) = \$100$ making 60 CDs.
   ANSWER: \$800

   (b) ANSWER: $P(x) = 15x - C(x) = -0.03x^2 + 18.6x - 208$.

   (c) HINT: The formula for profit is a parabola that opens downward. The maximum profit is the second coordinate of the vertex.
   ANSWER: \$2675

4. (a) HINT: Find the equation of the line that is Dory's path ($y = -\frac{1}{6}x - 10$) and the equation of the circle that bounds the clearing ($x^2 + y^2 = 3600$). Find where these two curves intersect. One point will be $(-60, 0)$. The other will be Dory's point of exit from the clearing.
   ANSWER: $(56.7568, -19.4595)$

   (b) HINT: Use the formulas for linear motion in Chapter 23 of the text.
   ANSWER: $x(t) = 3.8919t - 60$, $y(t) = -0.6486t$

   (c) HINT: Compute the distance across the clearing and divide by 20 seconds.
   ANSWER: Dory must run 5.9184 feet per second or faster.

5. (a) HINT: You're looking for the equation of the line through the points $(0, 220)$ and $(8, 609)$.
   ANSWER: $C(t) = 48.625t + 220$

(b) HINT: You're looking for the exponential function through the points $(0, 6)$ and $(8, 50)$.
ANSWER: either $B(t) = 6\left(\frac{25}{3}\right)^{t/8} = 6(1.303473914)^t$ or $B(t) = 6e^{0.265032942t}$

(c) HINT: Solve the equation $B(t) = 72$ for t ($t = 9.37584072$) and plug your result into $C(t)$.
ANSWER: $C(9.37584072) = 676$ coins

6. (a) ANSWERS: $V(17, 62)$ $W(24.5, 55)$

(b) ANSWERS: $A = 7$, $B = 10$ $C = 17$ or 7 or 27, etc., $D = 62$

7. (a) HINT: Find the angle between Rita's starting location and her location after 4 seconds. ($\theta = \cos^{-1}(\frac{59}{100}) = 0.9397$ radians) Then angular speed is $\omega = \frac{\theta}{t}$.
ANSWER: $\omega = 0.2349$ radians per second

(b) ANSWER: $v = 23.4934$ feet per second

(c) HINT: Use the formulas for circular motion from Chapter 22 of your text.
ANSWER: $x(t) = 100\cos(0.2349t - \frac{\pi}{2})$, $y(t) = 100\sin(0.2349t - \frac{\pi}{2}) + 103$