$\qquad$ Section: $\qquad$

| 1 | 10 |  |
| :---: | :---: | :--- |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| Total | 70 |  |

- Complete all questions.
- You may use a calculator during this examination. Other electronic devices are not allowed, and must be turned off for the duration of the exam.
- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator when an algebraic method is available, you will not receive full credit.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 170 minutes to complete the exam.

1. Beth is making an expedition in a hot air balloon. Because of atmospheric conditions, her altitude is a sinusoidal function of time. At 3 AM on March 1, she was at her minimum altitude: 2000 feet. Her altitude then increased, reaching her maximum altitude of 2500 feet at 7:30 AM on March 1.
On March 1, how much of the day was her altitude below 2100 feet?
2. In the year 1980, the city of Gumbolantenden had a population of 20,000. The population of Gumbolantenden grows at the rate of $1.07 \%$ each year.
The population of the city of Attalioto is growing exponentially. In the year 2000, the population was 40,000 . In the year 2005 , the population was 62,000 .
When will the populations of the two cities be the same? Express your answer in years after 1980.
3. Nell and Kit are moving in the $x y$-coordinate plane, with the dimensions of the plane in feet (so, for example, the point $(0,1)$ is 1 foot from the point $(0,0)$ ).
Nell starts from the point $(20,8)$ and heads directly toward the point $(1,4)$. She will take 3 seconds to get to the point $(1,4)$.
Kit starts from the point $(1,2)$ and travels along the line $y=-0.5 x+2.5$, heading toward the $y$-axis, at the rate of 5 feet per second.
(a) Find Nell's equations of motion. That is, express her location's $x$ - and $y$-coordinates as functions of time.
(b) Find Kit's equations of motion. That is, express his location's $x$ - and $y$-coordinates as functions of time.
4. Bob is walking directly from Albany to Birmingham. Birmingham is 10 km north and 7 km east of Albany.
Cambridge is 3 km east and 4 km south of Birmingham.
On Bob's walk, how close does he come to Cambridge?
5. Godzilla is attacking - you must flee! Before you flee, you will take two measurements of Godzilla. While Godzilla is an unknown distance away, you measure the angle up to the top of Godzilla's head to be $73^{\circ}$. You then run 120 feet further away, and measure the angle again: this time it is $66^{\circ}$. In the time between the measurements, Godzilla's height increased by 24 feet.
How tall was Godzilla at the time of the second measurement?
6. Dorritt is planning to make some enclosures for her llamas. She wants to make three identical square enclosures and a rectangular enclosure with its long side twice as long as its short side. For example, her enclosures might look like this:


She can afford to buy 300 feet of fencing. What should the dimensions of the square enclosures be to result in the maximum total area of all her enclosures?
7. Let $f(x)=2 x+5-4|x-1|$.
(a) (4 points) Write the multipart rule for $f(x)$.
(b) (6 points) Find all values of $a$ that satisfy the following equation:

$$
f(a)=7 a-8
$$

