1. A producer is figuring out how much to charge for tickets to a show. If she charges \$0 per ticket, she will make \$0. If she charges \$5 per ticket, she will make \$1775. If she charges \$15 per ticket, she will make \$975.

If the amount of money she makes is a quadratic function of the ticket price, what is the maximum possible amount of money she can make from the sale of the tickets?

$x =$ price per ticket

$f(x) =$ how much she makes. $= ax^2 + bx + c$

Know: 1) $f(0) = \$0 \Rightarrow a(0)^2 + b(0) + c = 0 \Rightarrow \boxed{c=0}$

2) $f(5) = 1775 \Rightarrow \boxed{25a + 5b + \cancel{c} = 1775}$

3) $f(15) = 975 \Rightarrow \boxed{225a + 15b + \cancel{c} = 975}$

After we eliminate $c=0$, we get 2 equations in a & b:

$$\begin{cases} 25a + 5b = 1775 \\ 225a + 15b = 975 \end{cases}$$

Divide both by 5:

$$\begin{cases} 5a + b = 355 \Rightarrow \boxed{b = 355 - 5a} \\ 45a + 3b = 195 \end{cases}$$

$\boxed{b = 500}$

$$45a + 3(355 - 5a) = 195$$

$$45a + 1065 - 15a = 195$$

$$3\cancel{0}a = -87\cancel{0}$$

$$a = \frac{-87}{3} \Rightarrow \boxed{a = -29}$$

So $\boxed{f(x) = -29x^2 + 500x}$ This is a quadratic, whose graph is concave-down $(a = -29 < 0)$. Hence it's maximal at vertex: $x = -\frac{b}{2a} = -\frac{500}{2(-29)} = \frac{250}{29} \cong 8.620689\ldots$

That is, she should charge $x \cong \$8.62$ /ticket.

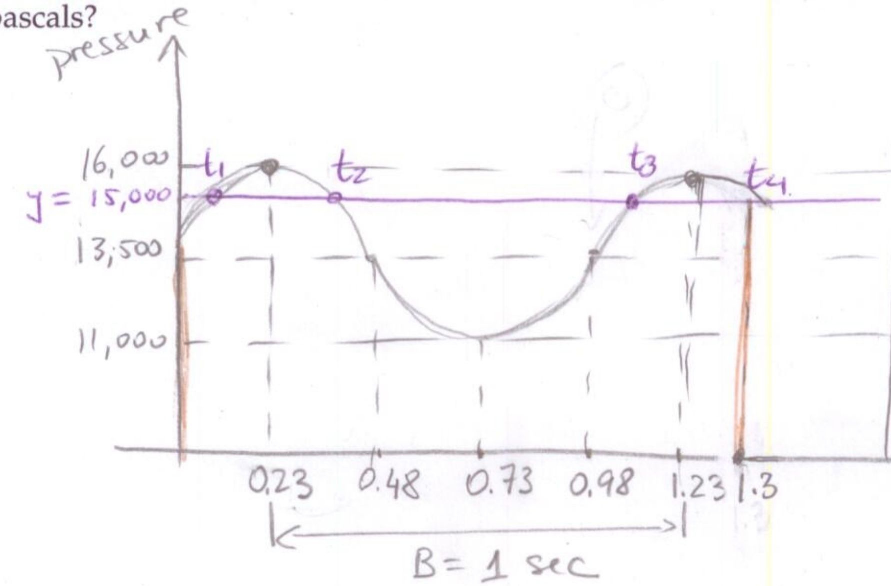
to make a max. of:

$$f(8.62) = -29(8.62)^2 + 500(8.62) = \boxed{\$2155.17}$$

2. The pressure inside an artery is varying sinusoidally. With a heart beat at a rate of 60 beats per minute, the period of this sinusoidal function is 1 second. The maximum pressure is 16000 pascals, and the minimum pressure is 11000 pascals.

   At time $t = 0$, you begin to measure the pressure. A maximum is attained for the first time at $t = 0.23$ seconds.

   Between $t = 0$ and $t = 1.3$, how much time (in seconds) is the pressure above 15000 pascals?



$A = \frac{16,000 - 11,000}{2} = 2,500$

$D = \frac{16000 + 11,000}{2} = 13,500$

$B = 1$ (given)

$C = x_{max} - B/4$

$= 0.23 - 0.25 = -0.02$

OR $1.23 - 0.25 = 0.98$

OR ...

t (sec.)

$$P(t) = 2,500 \sin\left(\frac{2\pi}{1}\underbrace{(t - 0.98)}_{\text{OR: } (t+0.02)}\right) + 13,500.$$

$$15,000 = 2,500 \sin(2\pi(t - 0.98)) + 13,500$$

$$\sin(2\pi(t - 0.98)) = \frac{15,000 - 13,500}{2,500} = 0.6$$

PS: $2\pi(t - 0.98) = \sin^{-1}(0.6)$

$t = \frac{\sin^{-1}(0.6)}{2\pi} + 0.98$

$\approx \boxed{1.08241638 = t_3}$ on graph

subtracting 1 period $B = 1$

$\boxed{t_1 = 0.08241638}$

SS: $2\pi(t - 0.98) = \pi - \sin^{-1}(0.6)$

$t = \frac{\pi - \sin^{-1}(0.6)}{2\pi} + 0.98$

$\approx \boxed{1.37758362\ldots = t_4}$

$\boxed{t_2 = t_4 - B \approx 0.3775836\ldots}$

note that t_4 is > 1.3 so we'll only go to 1.3.

The pressure is above the purple line of 15,000 Pa from t_1 to t_2 and from t_3 to 1.3

so total time $= (t_2 - t_1) + (1.3 - t_3)$

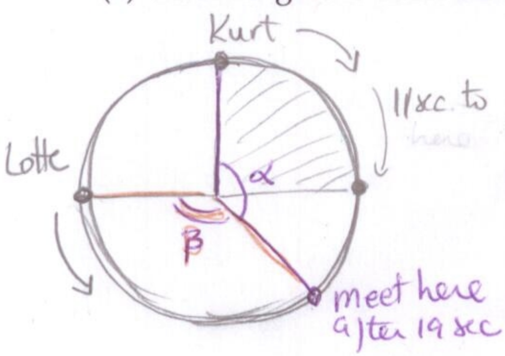
$\approx (0.295167) + (0.2175836) \approx$ 0.51275 sec

3. Kurt and Lotte are running around a circular track at constant speeds. They start running at the same time.

Kurt runs clockwise, and starts from the northernmost point. He takes 11 seconds to reach the easternmost point for the first time.

Lotte runs counterclockwise, starting from the westernmost point. She passes Kurt for the first time after 19 seconds.

(a) How long does Lotte take to complete one lap of the track?



11 sec. to go $\frac{\pi}{2}$ rad $\Rightarrow \omega_K = \dfrac{\pi/2 \text{ rad}}{11 \text{ sec}} = \dfrac{\pi}{22} \dfrac{\text{rad}}{\text{sec}}$ (neg. dir!)

In 19 sec Kurt travels an angle α:

$$\alpha = \omega_K (19) = \frac{19\pi}{22} \text{ radians.}$$

In 19 sec Lotte travels an angle β:

$$\beta = \frac{3\pi}{2} - \alpha = \frac{3\pi}{2} - \frac{19\pi}{22} = \frac{33\pi - 19\pi}{22} = \frac{14\pi}{22}\text{rad} = \frac{7\pi}{11}\text{rad}$$

So: $\beta = \omega_L(19) = \frac{7\pi}{11}\text{rad} \Rightarrow \omega_L = \dfrac{7\pi}{(11)(19)} = \dfrac{7\pi}{209}$ rad/sec.

$1 \text{ lap} = 2\pi \text{ rad} = \omega_L t \Rightarrow t = \dfrac{2\pi \text{ rad}}{\omega_L} = \dfrac{2\pi}{\left(\frac{7\pi}{209}\right)} = \dfrac{418}{7} \cong \boxed{59.714 \text{ seconds}}$

(b) Suppose the track has a radius of 20 meters. Find the distance between Lotte and Kurt after they have been running for 25 minutes.

After 25 min = 1500 sec

KURT

Kurt's coord: $X(1500) = 20\cos\left(\frac{\pi}{2} - \frac{\pi}{22}(1500)\right) \cong 10.8128$

$Y(1500) = 20\sin\left(\frac{\pi}{2} - \frac{\pi}{22}(1500)\right) \cong 16.825$

$\Rightarrow \boxed{(10.8128, 16.825)}$

LOTTE

Lotte's coordinates:

$X(1500) = 20\cos\left(\pi + \frac{7\pi}{209}(1500)\right) \cong -14.61226$

$Y(1500) = 20\sin\left(\pi + \frac{7\pi}{209}(1500)\right) \cong 13.65584$

$\Rightarrow \boxed{(-14.612, 13.6558)}$

distance is:

$$\cong \sqrt{(10.8128 - (-14.612))^2 + (16.825 - 13.6558)^2}$$

$$\cong \sqrt{(25.425)^2 + (3.169)^2} \cong \boxed{25.622 \text{ meters}}$$

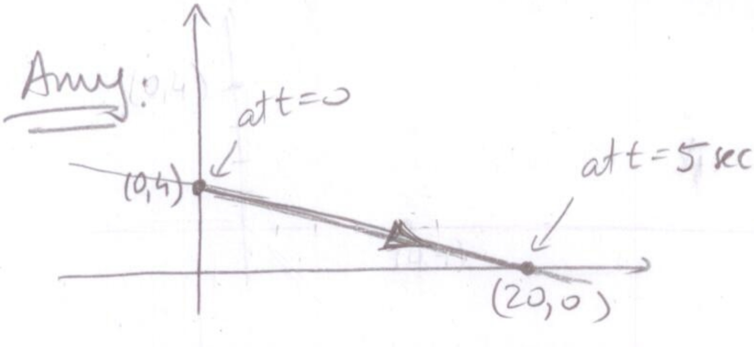
4. Amy and Hans are moving at constant speeds along lines in the xy-plane.

Amy starts at $(0, 4)$ and moves along the line $y = 4 - 0.2x$. She will reach the x-axis in 5 seconds.

$0 = 4 - 0.2x \Rightarrow x = 4/0.2 = 20 \leftarrow$ x intercept

Hans starts at the same time as Amy. He starts from the origin, heads directly toward the point $(8, 12)$, and will reach it in 4 seconds.

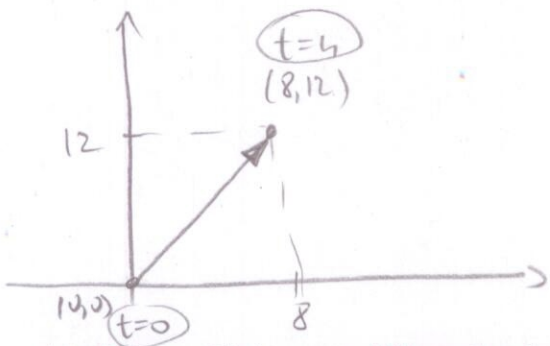
(a) Give Amy's equations of motion (i.e., her x- and y-coordinates as functions of time, t).



$v_x = \frac{\Delta x}{\Delta t} = \frac{20}{5} = 4, \quad v_y = \frac{\Delta y}{\Delta t} = \frac{-4}{5}$

$$\boxed{\begin{cases} x_A(t) = 4t & (\leftarrow x_0 + v_x t) \\ y_A(t) = 4 - \frac{4}{5}t & (\leftarrow y_0 + v_y t) \end{cases}}$$

(b) Give Hans' equations of motion (i.e., his x- and y-coordinates as functions of time, t).



$v_x = 8/4 = 2, \quad v_y = \frac{12}{4} = 3$

$$\boxed{\begin{cases} x_H(t) = 2t \\ y_H(t) = 3t \end{cases}}$$

(c) When will Amy and Hans be closest together?

Distance between Amy and Hans at t sec is:

$$d = \sqrt{(x_A - x_H)^2 + (y_A - y_H)^2} = \sqrt{(2t)^2 + (4 - 4/5\,t - 3t)^2}$$

$$= \sqrt{4t^2 + \left(4 - \frac{19t}{5}\right)^2} = \sqrt{4t^2 + (4 - 3.8t)^2} = (\dots)$$

$$= \sqrt{18.44t^2 - 30.4t + 16}$$

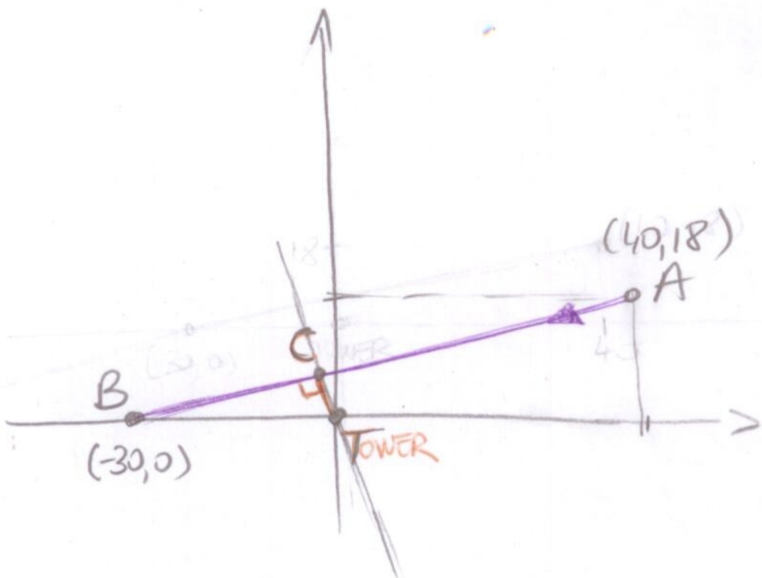
distance is smallest when $18.44t^2 - 30.4t + 16$ is smallest

This is a concave-up parabola $(a = 18.44 > 0)$, so it's smallest at its vertex $t = \frac{30.4}{2(18.44)} \cong 0.824295$

They are closest at $\cong \boxed{0.824295 \text{ seconds}}$

5. Bob is 40 km east and 18 km north of a radio tower. Bob walks in a straight line toward a point 30 km due west of the tower, and then continues in the same direction on to his destination.

   How close does Bob come to the radio tower on his walk?



Line of motion (AB) is

$y = mx + b$

where $m = \frac{\Delta y}{\Delta x} = \frac{18-0}{40-(-30)}$

$= \frac{18}{70} = \frac{9}{35}$

Plug in coord. of B:

$0 = \frac{9}{35}(-30) + b$

$b = 54/7$

line AB: $\boxed{y = \frac{9}{35}x + \frac{54}{7}}$

Line TC is $\perp$ to AB so $m_{TC} = -\frac{1}{9/35} = -\frac{35}{9}$

and its y-intercept is zero

line TC: $\boxed{y = -\frac{35}{9}x}$

Point C is at intersection, so $-\frac{35}{9}x = \frac{9}{35}x + \frac{54}{7}$

$x\left(\frac{-35}{9} - \frac{9}{35}\right) = \frac{54}{7}$

point $\underline{\underline{C}}$ $\begin{cases} x_C \cong -1.8606431\ldots \\ y_C \cong -\frac{35}{9}(-1.86) \cong 7.2358\ldots \end{cases}$

distance CT $= \sqrt{(x_c - 0)^2 + (y_c - 0)^2}$

$\cong \sqrt{(-1.8606)^2 + (7.2358)^2}$

$\cong \boxed{7.47 \text{ km}}$

6. The more hours Matt puts into his training, the faster he will be able to complete an upcoming 200 km bicycle ride. If he puts in 30 hours of training, he will be able to complete it in 9 hours. If he trains for 40 hours, he will be able to complete the ride in 8 hours. No matter how much he trains, he will never be able to complete the ride in less than 6 hours.

Assume that the time Matt will take to complete the ride is a linear-to-linear rational function of the time he spends training.

If Matt wants to finish the ride in 8.5 hours, how much training should he do?

x = time he spends training

$f(x)$ = time to complete the ride

$$f(x) = \frac{ax+b}{x+d}.$$

Know:

$$\begin{cases} f(30)=9 \Rightarrow \dfrac{30a+b}{30+d} = 9 \Rightarrow 30a+b = 270+9d. & (1) \\ f(40)=8 \Rightarrow \dfrac{40a+b}{40+d} = 8 \Rightarrow 40a+b = 320+8d & (2) \\ \text{Horiz. asymptote: as } x \to \infty,\ f(x) \to 6 \Rightarrow \boxed{a=6} & (3) \end{cases}$$

Replacing $a=6$ into (1) & (2), and solving for b:

$$\left.\begin{cases} b = 270+9d-30(6) = 9d+90 \\ b = 320+8d-40(6) = 8d+80 \end{cases}\right\} \Rightarrow 9d+90 = 8d+80$$

$$\boxed{d=-10}$$

$$b = 8(-10)+80$$

$$\boxed{b=0}$$

$$\boxed{f(x) = \frac{6x}{x-10}}$$

$$8.5 = \frac{6x}{x-10}, \quad \text{solve for } x$$

$$8.5x - 85 = 6x$$

$$2.5x = 85$$

$$x = 85/2.5 = 34$$

$\boxed{\text{He should train 34 hrs.}}$

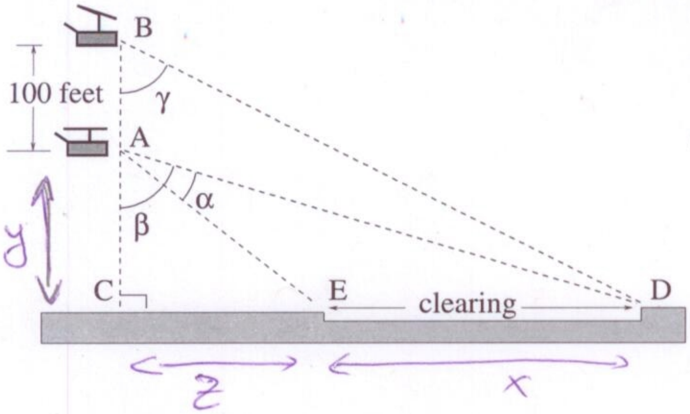
7. The crew of a helicopter needs to land temporarily in a forest. They spot a flat horizontal piece of ground (a clearing in the forest) as a potential landing site, but are uncertain whether it is wide enough. They make two measurements from point A (see figure) and find $\alpha = 19^o$ and $\beta = 60^o$.

They then rise vertically 100 feet to point B and measure $\gamma = 53^o$.

Determine the width of the clearing.

Label distances as in picture.

We'll use the 3 right triangles:



1) ΔACE: $\tan(\beta - \alpha) = \frac{z}{y}$.

$\tan(41^\circ) = z/y$

$\Rightarrow \boxed{z = y \tan(41^\circ)}$ ①

2) ΔACD: $\tan\beta = \frac{x+z}{y} \Rightarrow \boxed{x+z = y\tan(60^\circ)}$ ②

3) ΔBCD: $\tan\gamma = \frac{x+z}{100+y} \Rightarrow \boxed{x+z = (100+y)\tan(53^\circ)}$ ③ $\Rightarrow$

$\Rightarrow y\tan(60^\circ) = (100+y)\tan(53^\circ)$

$y \cdot (\tan 60^\circ - \tan 53^\circ) = 100 \tan 53^\circ$

$y = \frac{100 \tan 53^\circ}{\tan 60^\circ - \tan 53^\circ} \cong \boxed{327.66 \text{ feet} = y}$

make sure your calc. is in degree mode.

Replace in ①: $z \cong 327.66 \tan(41^\circ) \cong \boxed{284.83 \text{ feet} = z}$

Replace y & z in ②: $x \cong 327.66 \tan(60^\circ) - 284.83$

$\boxed{x \cong 282.69}$

The clearing is $\cong$ 282.69 feet wide.