Math 120 - Winter 2014

Final Exam

March 15, 2014

Name: ______________________ Student ID no. : ______________

Signature: ______________________ Section: ______________

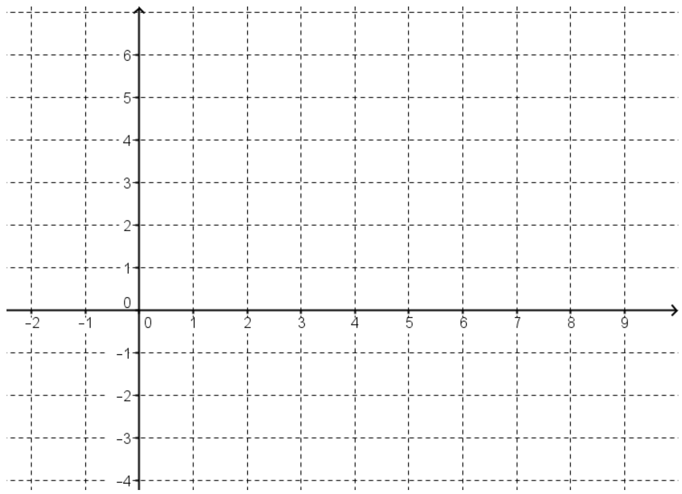
1	10	
2	12	
3	10	
4	8	
5	8	
6	10	
7	10	
8	12	
Total	80	

- In addition to this cover page, your exam should contain 8 problems on 8 pages. When the test starts, **check that you have a complete exam!**
- Unless otherwise stated, you must **show how you get your answers**, and use the methods learned in this class. Answers with incomplete or no supporting work will result in little or no credit, even if correct. If you use a trial-and-error method when an algebraic method is available, you will not receive full credit.
- You may use a scientific calculator during this examination. Graphing calculators are not allowed. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam. Please TURN OFF your cell phone now.
- Please box your final answer. You may either round off your final answer to 2 or more decimal digits, or leave it in exact form (for instance, $\sqrt{2\pi}+5$, or 7.51). Do not round any intermediate values or bases of exponential functions.
- You may use one hand-written 8.5 by 11 inch page of notes. You may not share notes or use any other papers. All work should be your own.
- You have 2 hours and 50 minutes to complete the exam. Good luck!

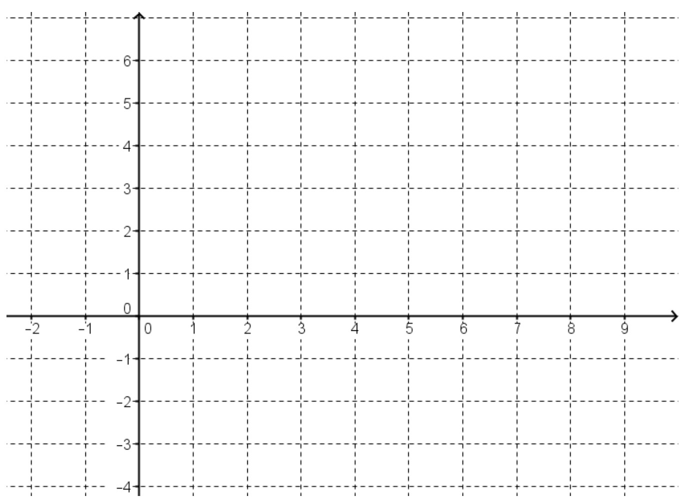
1. [10 points] Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} -2x + 7, & \text{if } x < 2 \\ 1 + \sqrt{4 - (x-2)^2}, & \text{if } 2 \leq x \leq 4 \\ -x + 5 & \text{if } x > 4 \end{cases}$$

(a) Sketch the graph of $y = f(x)$



(b) Sketch the graph of $y = f(x+2) - 1$



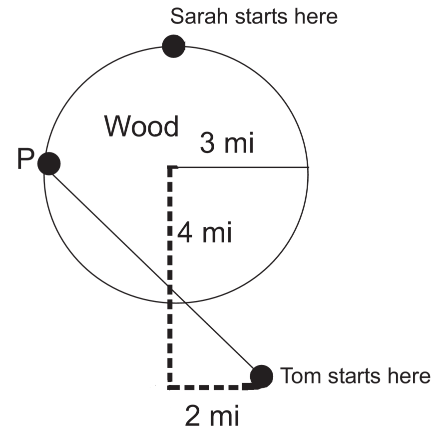
(c) Let $g(x) = e^x$, and let $f(x)$ be the function defined at the top of the page.

Compute the value $f(g(0))$

(d) Compute the value of $f^{-1}(4.5)$, where $f(x)$ is the function defined at the top of the page.

2. **[12 points]** In the Redwood park there is a wood in the shape of a circle of radius 3 miles. At time $t = 0$ hours, Sarah starts running around the wood in counterclockwise direction, at an angular velocity $\omega = 2$ rad/hr, starting from a point 3 miles due North of the center of the circle.

At the same time time ($t = 0$) Tom starts running at a uniform speed of 7 mph, starting from a point 2 miles East and 4 miles South of the center of the circle, and running in a straight line path towards a point P located 3 miles due West of the center of the circle. (see picture)



(a) Who will reach point P first ?
Show work – no credit for unjustified answers.

(b) Impose a coordinate system with the origin at the center of the circle. Find the parametric equations of motion for Tom. That is, find his coordinates $x(t)$ and $y(t)$ at t hours.

3. [10 points] At the start of a biology experiment there are 2,000 cells in a Petri dish.

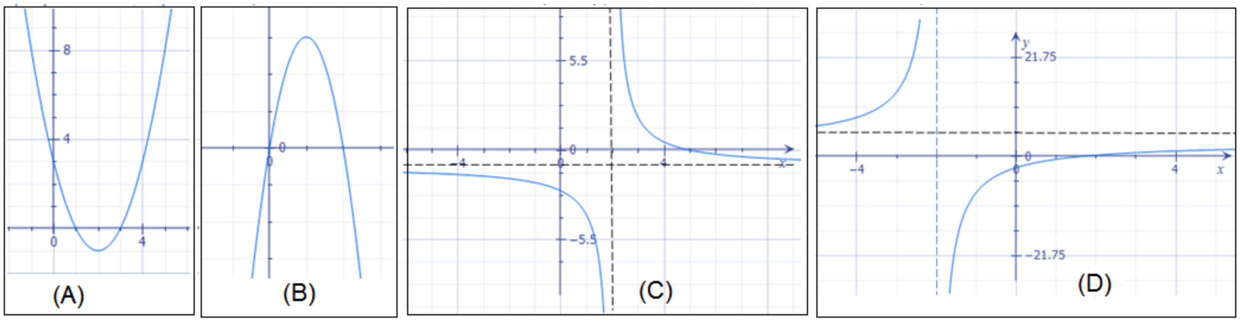
After 1 minute, there are 3,000 cells in that Petri dish.

(a) Assuming that the number of cells grows exponentially, how long does it take to double ? Show your steps, and include units in your answer.

(b) When the number of cells reaches 10,000, it no longer grows exponentially and instead it grows at a constant rate of 500 cells per minute from then on. Write a formula for the multi-part function $N(t)$ that gives the number of cells in the Petri dish at all times $t \geq 0$.

4. [8 points]

(a) Consider these four graphs, labeled (A)-(D).



For each of the following functions, list the letter of the graph that corresponds to it, or state NONE if the function does not match any of the graphs.

(a) $f(x) = \dfrac{3x-5}{x+2}$ corresponds to the graph labeled:

(b) $g(x) = -x^2 + 4$ corresponds to the graph labeled:

(c) $h(x) = \dfrac{-x+5}{x-2}$ corresponds to the graph labeled:

(d) $k(x) = x^2 - 4x + 3$ corresponds to the graph labeled:

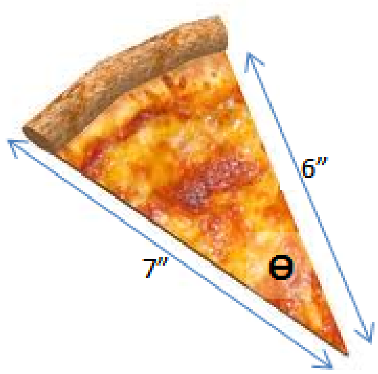
(b) Solve the following equation:

$$\ln\left(\frac{3x-2}{x-7}\right) = 0$$

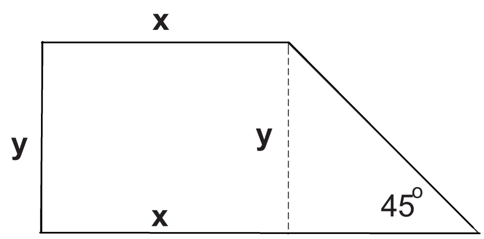
5. **[8 points]** At Papa Johns, a Large pizza has a **diameter** of 14 inches, and an Extra Large pizza has a **diameter** of 16 inches. Suppose each slice of pizza is cut normally (wedge-shaped, with the vertex at the center of the pizza pie.)

(a) Which slice would be larger: a slice of a Large pizza cut at a $45°$ angle, or a slice of an Extra Large pizza cut at an angle of $\frac{\pi}{5}$ radians? (show work - no credit for unjustified answers.)

(b) You order a Large pizza, uncut. You like the plain crust portion the most. If the toppings part of the pizza has a 6 inch radius, at what minimum angle θ should you cut your slice in order to get at least 3 square inches of plain crust per slice? Give your answer in degrees.



6. [10 points] You have 100 cm of wire to enclose a trapezoidal shape like in the figure below.



(a) Write the area of the trapezoidal shape in terms of y only.

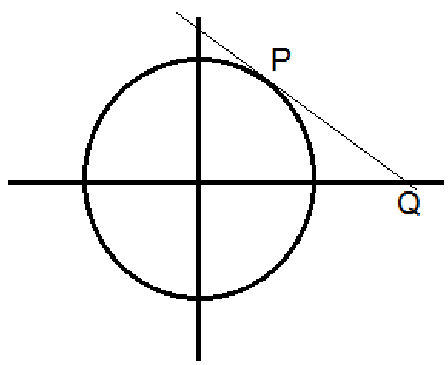
(Recall that the area of a trapezoid with parallel sides b_1 and b_2 and height h is $A = \frac{1}{2}(b_1 + b_2)h$ *)*

(b) Find the dimension y that maximizes the area of the trapezoid you can enclose.

7. [10 points] An object rotates counterclockwise at 1 revolution every 6 seconds around a circle of radius 2 centered at the origin. It starts at $t = 0$ seconds at the topmost point, $(0, 2)$.

(a) Find the coordinates of the object after 5 seconds. Round your answer to two decimal digits.

(b) The rotating object from above emits a beam of light along the line tangent to the circle. Find the point Q where the beam of light interesects the x-axis at $t = 5$ seconds.
(Recall that the line tangent to the circle at a point P is perpendicular to the radial line at P.)

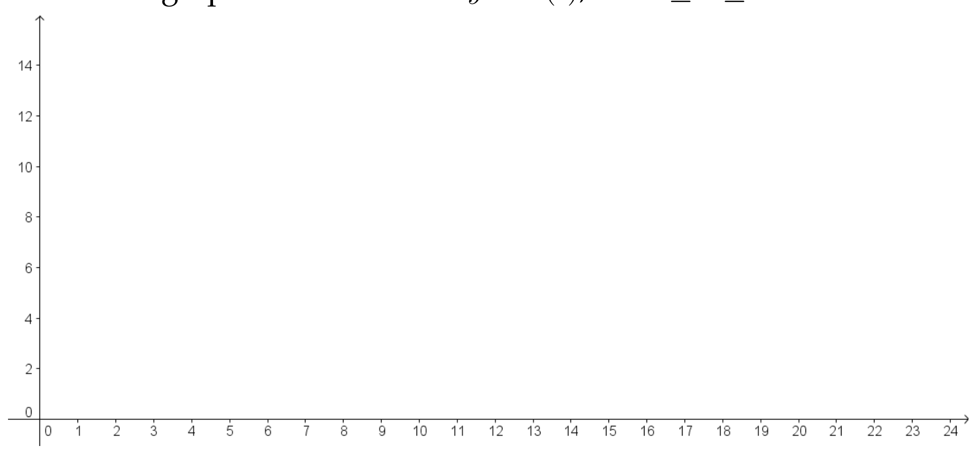


8. [12 points] The predicted times and heights of the high and low tides for the seaside village of Portwenn during a certain day are:

Time of day	Low/High Tide Height (in meters)
00:30	4.8
06:30	14.4
12:30	4.8
18:30	14.4

(a) Find a sinusoidal function in standard form, $h(t) = A\sin(\frac{2\pi}{B}(t - C) + D$, which models the tide height data for Portwenn at t hours past midnight, on the given day.

(b) Sketch the graph of the function $y = h(t)$, for $0 \leq t \leq 24$ hours.



(c) A boat requires a tide height of 10 meters or more to be able to enter a harbor. Compute all the time intervals during this day when the boat could enter the Portwenn harbor.