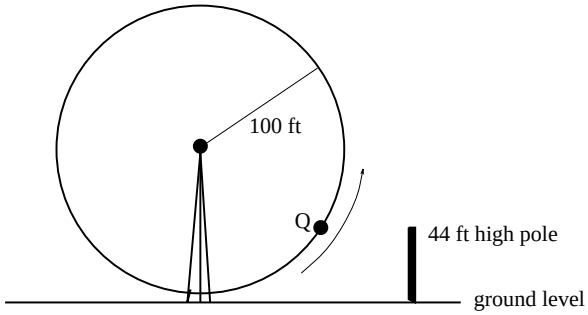
HOMEWORK (Winter/Spring), Week 1

1. Stewart, section 10.1: #5, 6, 9, 16. (Make sure you have studied the parametric equations supplementary reading first, or alternatively, read section 10.1).

2. A ferris wheel of radius 100 feet is rotating at a constant angular speed ω counterclockwise. Using a stopwatch, the rider finds it takes 3.4 seconds to go from the lowest point on the ride to a point Q, which is level with the top of a 44 ft pole. Assume the lowest point of the ride is 3 feet above ground level.



   Let $Q(t) = (x(t), y(t))$ be the coordinates of the rider at time t seconds; i.e., the parametric equations.

   (a) Find the formulas for $x(t)$ and $y(t)$, assuming the rider begins at the lowest position on the ride.. *(Note: Make sure you have looked over the websupplement "parametric equations.pdf").*

   (b) During the first revolution of the wheel, find the times when the rider's height above the ground is 80 feet. Give two decimal places of accuracy.

3. Let a, b be positive real numbers. Suppose a particle is moving in the xy plane according to the parametric equations $x(t) = a\cos(\pi t)$ and $y(t) = b\sin(\pi t)$.

   (a) Plot the position of the particle at times $t = \frac{k}{4}, k = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

   (b) Verify that if the particle is at position (x, y), then

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

   Note: this shows the particle is moving around the ellipse described by this equation.

   (c) What happens when $a = b$?

4. Stewart, section 2.1: #1, 5, 7.

5. If an object is dropped from an 80-meter high window, its height y above the ground at time t seconds is given by the formula $y = f(t) = 80 - 4.9t^2$. (Here we are neglecting air resistance.) Find the average velocity of the object between

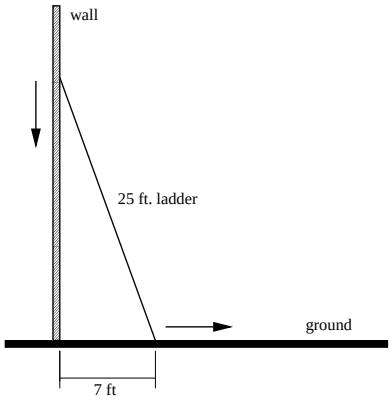
   (a) 1 sec and 1.1 sec,

   (b) 1 sec and 1.01 sec

   (c) 1 sec and 1.001 sec.

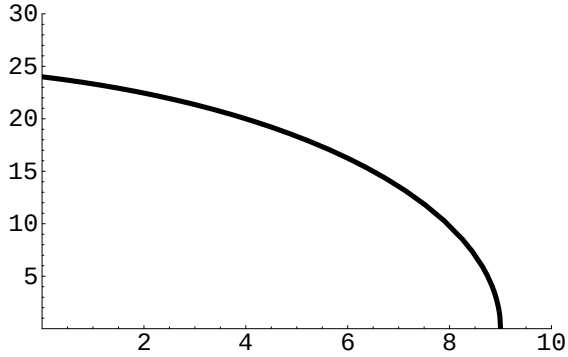
   Now use algebra to find a simple formula for the average velocity of the falling object between 1 sec and $1 + \Delta t$ sec. Determine what happens to this average velocity as Δt approaches 0. That is the instantaneous velocity at time $t = 1$ sec (it will be negative, since the object is falling).

6. A ladder 25 feet long is leaning against the wall of a building. Initially, the foot of the ladder is 7 feet from the wall. The foot of the ladder begins to slide at a rate of 2 ft/sec, causing the top of the ladder to slide down the wall. The location of the foot of the ladder at time t seconds is given by the parametric equations $(7 + 2t, 0)$. *(Note: Make sure you have looked over the websupplement "parametric equations.pdf").*



   (a) The location of the top of the ladder will be given by parametric equations $(0, y(t))$. Find the formula for $y(t)$. What is the domain of t values?

   (b) The graph of the function $y(t)$ is given below. Compute the average velocity of the top of the ladder on these time intervals: $[0, 2]$, $[2, 4]$, $[6, 8]$, $[8, 9]$. Indicate what the average velocity is telling you in terms of the picture.



   (c) The foot of the ladder is moving at a constant rate; how about the top of the ladder?

7. Stewart, section 2.2: #1, 7, 9, 10, 13, 15, 21, 27, 34(a), 40.

8. Stewart, section 2.3: #1, 11,12,20,29,35,37,48,62.