HOMEWORK, Week 3

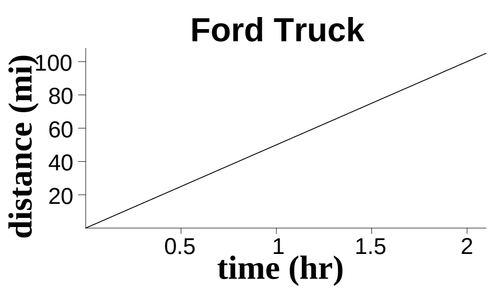
1. Stewart, section 2.3: #1, 11, 12, 15, 20, 29, 35, 37, 52, 59.

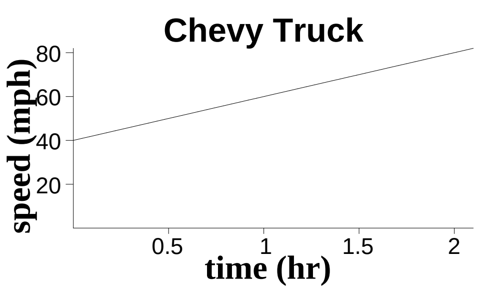
2. Stewart, section 2.5: #4, 15, 17, 19, 31, 33, 37, 41.

3. Stewart, section 2.6: #4, 11, 19, 27, 55(a).

4. Stewart, section 2.7: #3, 7, 10, 11, 15, 17, 19.

5. Two trucks (a Ford and a Chevy) pass through an intersection at the same time. The lefthand graph below relates time (in hours) and distance (in miles) for the Ford truck, beginning the instant it passes through the intersection. The righthand graph below relates time (in hours) and speed (in mph) for the Chevy truck, beginning the instant it passes through the intersection. In both cases, these quantities are related by linear functions; what are the rules for these functions? What does the slope of each line tell us? Which vehicle is going faster at time $t = 15$ minutes?





6. (a) Use a table of values to estimate $\lim_{x\to\infty}(1+\frac{1}{x})^x$ to 4 decimal places.

   (b) Use a table of values to estimate $\lim_{x\to\infty}(1+\frac{2}{x})^x$ to 4 decimal places.

   (c) Use a table of values to estimate $\lim_{x\to\infty}(1-\frac{1}{x})^x$ to 4 decimal places.

   (d) On your calculator, compute the numbers e, e^{-1}, e^2 and relate these to (a)-(c). What would you guess to be $\lim_{x\to\infty}(1+\frac{m}{x})^x$ (m is a constant)?

7. Tafu is working with subatomic particles in the Physics lab. A positron is traveling in a straight line down the particle accelerator. Tafu models its position with the function $p(t) = \dfrac{t^2}{t-1}$ where $t > 1$.
   (a) Compute $\lim\limits_{t\to\infty} p(t)$. Where does the positron eventually go? Compute $\lim\limits_{t\to 1^+} p(t)$. Where is the positron coming from?
   (b) Just for fun, also compute $\lim\limits_{t\to 1^-} p(t)$.

(c) Let $a > 1$ be some fixed number. Let $h > 0$ be some small number. Write out a formula for the slope of the secant line to the graph of $x = p(t)$ between the point at $t = a$ and the point at $t = a + h$.

(d) Keep a fixed and think of your formula in (c) as a function in the variable h. Call it $f(h)$. Simplify your formula for $f(h)$ as much as possible and then compute $\lim_{h\to 0} f(h)$.

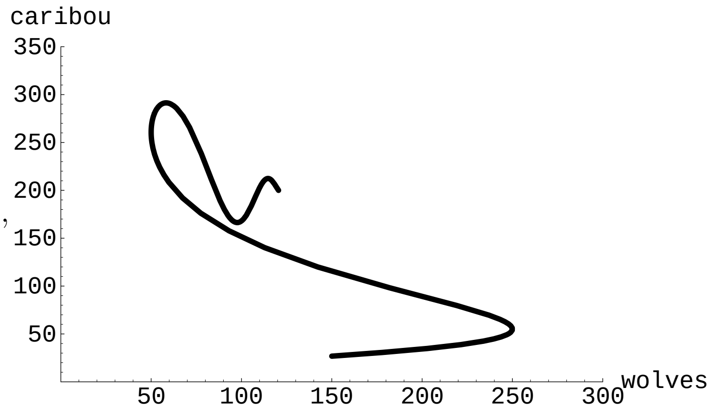
(e) Find a time $t = a$ when the positron's speed is zero. This is when it turns around. Where is it located at this time?

8. The population of caribou and wolves in an isolated alaskan valley are modeled by the equations:

$$w(t) = 100\sin\left(\frac{4\pi t}{2t+1}\right) + 150;$$

$$c(t) = 200e^{-t/3}\sin\left(\frac{\pi}{3}(t-1)\right) + 200,$$

where t represents years after the year 1960. We can plot the points $(w(t), c(t))$ in a coordinate system. Here is the graph of all points when $0 \le t \le 10$



(a) Find the population of each species at times $t = 0, 0.5, 1, 5, 10$ years.

(b) When will the population of wolves be a maximum? What is the caribou population at this time and where is this on the plot?

(c) When will the population of wolves be a minimum? What is the caribou population at this time and where is this on the plot?

(d) What is the limiting population of both species in this ecosystem?

9. The location of an object on the $x-$axis at time t seconds is given by $x(t) = a + mt$, where a, m are constants.

(a) Where is the object located at time $t = 1$?

(b) What is the average velocity of the object between 1 and 2 seconds?

(c) What is the instantaneous velocity of the object at time $t = 1$?

(d) What is the instantaneous velocity of the object at time t?

10. The linear motion of an object begins at the location $(2, 3)$ and has parametric equations $(x(t), y(t))$.

(a) If the horizontal instantaneous velocity $v_x(t) = 1$ and the vertical instantaneous velocity $v_y(t) = 0$, sketch the path the object follows.

(b) If the horizontal instantaneous velocity $v_x(t) = 0$ and the vertical instantaneous velocity $v_y(t) = 1$, sketch the path the object follows.

(c) If the horizontal instantaneous velocity $v_x(t) = 1$ and the vertical instantaneous velocity $v_y(t) = 1$, sketch the path the object follows.

(d) If the horizontal instantaneous velocity $v_x(t) = 1$ and the vertical instantaneous velocity $v_y(t) = -1$, sketch the path the object follows.

(e) If the horizontal instantaneous velocity $v_x(t) = -1$ and the vertical instantaneous velocity $v_y(t) = -1$, sketch the path the object follows.