

Math 124 Homework (winter/spring) #4 Answers

4. $y = \frac{-x}{2\sqrt{3}} + \frac{2}{\sqrt{3}}$.

5. $\frac{112}{\sqrt{130}}$ ft/sec.

6. (a) $(.18)(.956)(200)e^{-0.18t} = 34.416e^{-0.18t}$

(b) $t = 0$: 34.42 cm/yr; $t = 1$: 28.75 cm/yr; $t = 6$: 11.69 cm/yr.

(c) 9.7042 yrs

7. Suppose the coordinates of the points are $P = (a, a^3)$ and $Q = (b, b^3)$. The slope of the tangent lines at P and Q are $3a^2$ and $3b^2$ respectively, so we must show that $b^2 = 4a^2$. The tangent line at P is given by the equation $y - a^3 = 3a^2(x - a)$. This line passes through Q , so $b^3 - a^3 = 3a^2(b - a)$. Factorizing, this gives $(b - a)(b^2 + ab + a^2) = 3a^2(b - a)$, so $(b - a)(b^2 + ab + a^2 - 3a^2) = 0$. We are assuming that $P \neq Q$, so $a \neq b$, so $(b - a) \neq 0$. Hence $0 = b^2 + ab - 2a^2 = (b - a)(b + 2a)$. So, we now have $b = -2a$, which implies that $b^2 = 4a^2$, as required.