HOMEWORK (winter/spring) #6

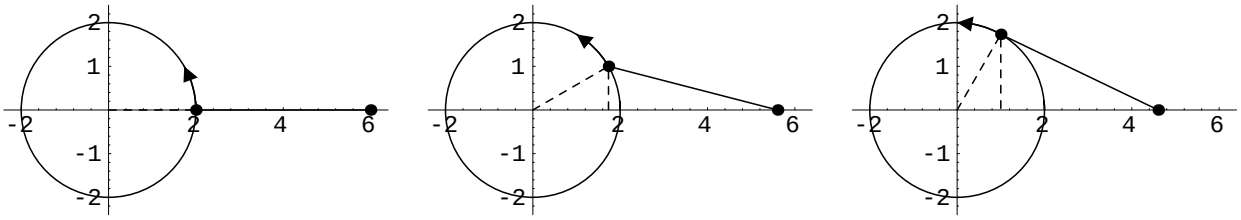
1. Stewart, section 3.6: #3,5,9,11,25,33,41,42,47,50.

2. A 4-centimeter rod is attached at one end A to a point on a wheel of radius 2 cm. The other end B is free to move back and forth along a horizontal bar that goes through the center of the wheel. At time $t = 0$ the rod is situated as in the diagram at the left below. The wheel rotates counterclockwise at 3 1/2 rev/sec. Thus, when $t = 1/21$ sec, the rod is situated as in the diagram at the right below.

   (a) How far is the right end of the rod (the point B) from the center of the wheel at time $t = 1/21$ sec?

   (b) Express the position of the right end of the rod as a function of t.

   (c) Express the speed of the right end of the rod as a function of t.



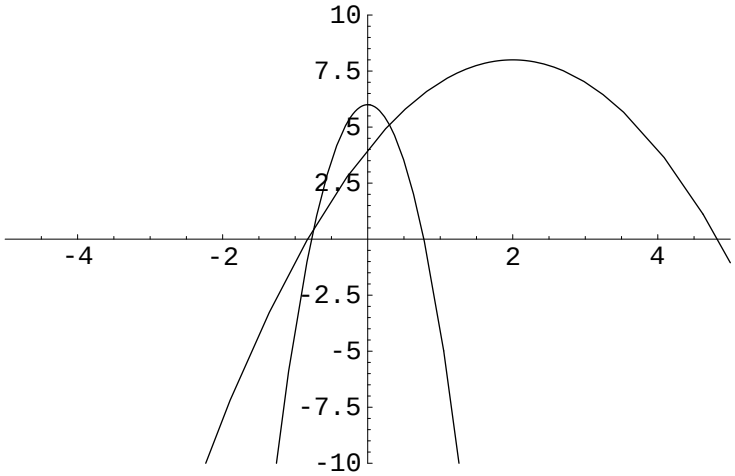
3. Sketch the graphs of the quadratic functions

$$f(x) = 6 - 10x^2$$

and

$$g(x) = 8 - (x - 2)^2,$$

are provided below. Find the lines simultaneously tangent to both graphs;i.e. find the equations of all such lines.



4. An object is moving around an ellipse according to the parametric equations

$$\begin{aligned} x(t) &= 2\cos(2\pi t) \\ y(t) &= \sin(2\pi t), \end{aligned}$$

where t has units of seconds and the coordinate axes have units of feet. The location of the object at time t will be $P(t) = (x(t), y(t))$.

(a) Sketch the ellipse of motion and compute the coordinates of $P(0), P(1/8)$, $P(1/2)$ and $P(7/8)$.

(b) Compute the horizontal velocity $x'(t)$ and vertical velocity $y'(t)$. What are the maximium (minimum) vertical and horizontal velocities?

(c) During the first trip of the object around the ellipse, find where the object is located when it has vertical velocity 3 ft/sec.

(d) During the first trip of the object around the ellipse, how much of the time will the horizontal velocity exceed 8 ft/sec?

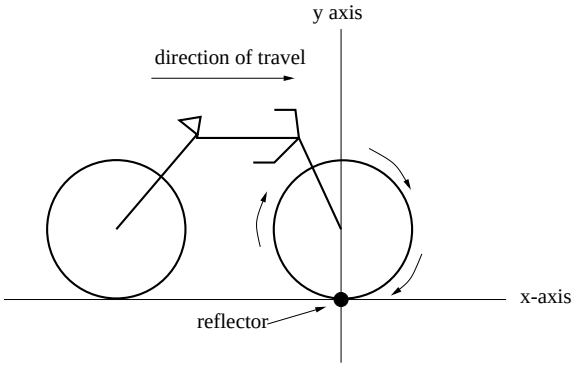
(e) Using the discussion in 10.2 of the textbook, find the equation of the tangent lines at each of the positions in (a).

5. Here are the parametric equations of a spiral curve passing through the point $(1, 0)$:

$$\begin{aligned} x(t) &= t^b \cos(2\pi t) \\ y(t) &= t^b \sin(2\pi t) \end{aligned}$$

For what value of b will the tangent line to the curve through the point $(1, 0)$ be $y = 5x - 5$.

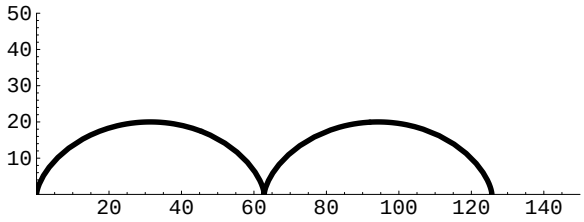
6. A bicycle rider is moving along a flat level road. A reflector is located on the rim of the front wheel; assume the wheel has radius 10 inches and the bike is moving at a rate of 20π in/sec in the direction indicated. As the bike moves from left to right in the picture below, the reflector will trace out a path in the imposed coordinate system:



The parametric equations for this motion are

$$\begin{aligned} x(t) &= 20\pi t + 10\cos(-2\pi t - \pi/2) \\ y(t) &= 10 + 10\sin(-2\pi t - \pi/2) \end{aligned}$$

where t is in units of seconds and the coordinate system has units of inches on the axes. A portion of the graph is pictured below for $0 \le t \le 2$:



(a) Compute the horizontal and vertical velocities of the motion of the reflector; i.e., $x'(t)$ and $y'(t)$.

(b) Compute $x'(t)$ and $y'(t)$ at $t = 0, 0.25, 0.5, 0.75, 1$ seconds. Indicate the location of the reflector on the graph at each of these times.

(c) When will the horizontal velocity be a maximum (minimum)? What is the maximum (minimum) horizontal velocity? Where is the reflector in the picture when this happens?

(d) When will the vertical velocity be a maximum (minimum)? What is the maximum (minimum) vertical velocity? Where is the reflector in the picture when this happens?

(e) The speed of the moving reflector at time t is defined by the equation

$$s(t) = \sqrt{[x'(t)]^2 + [y'(t)]^2}.$$

Compute $s(t)$. During the first 2 seconds, how many times will the speed be equal to 50 in/sec?