Math 124 Final Examination Spring 2008

Print Your Name	Signature
Student ID Number	**Quiz Section**
Professor's Name	**TA's Name**

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	9	
2	12	
3	15	
4	10	

Problem	Total Points	Score
5	12	
6	12	
7	10	
8	10	
9	10	
Total	100	

1. (9 points) Compute the following limits. You must show your work or adequately explain your answers.

(a) $\lim\limits_{x \mapsto 0} \dfrac{\tan(\pi\, x)}{\ln(1+x)}$

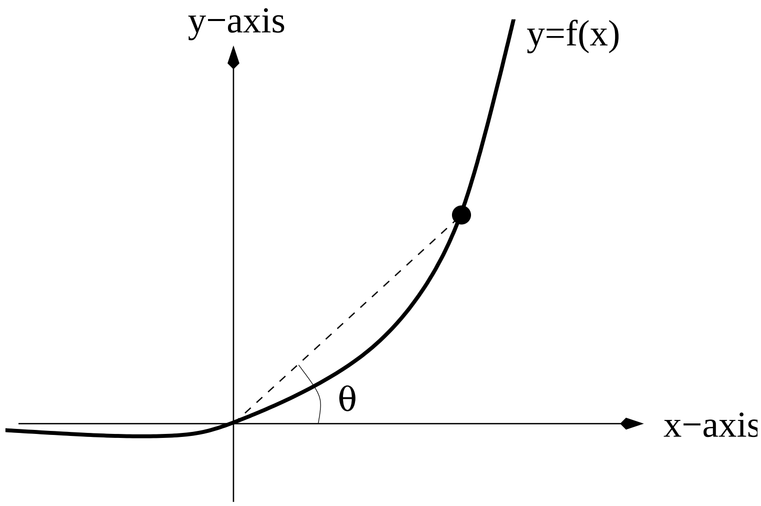
(b) $\lim\limits_{x \mapsto \frac{\pi}{2}^-} (\tan(x))^{\cos(x)}$

(c) $\lim\limits_{x \mapsto +\infty} \dfrac{\sqrt{100\, x^2+1} - x}{x-1}$

2. (12 points) A particle is moving on the curve

$$y = f(x) = x\, e^{x-1}\,.$$

Draw a line connecting the origin and the position of the particle. The angle θ pictured below is changing as the particle moves on the curve. When $\theta = \frac{\pi}{4}$ the rate of change of θ is 0.2 rad/sec. How fast is the x-coordinate of the particle changing at that time.



3. (15 points) Consider the function

$$f(x) = \frac{2}{3}x^{\frac{1}{2}} - x^{\frac{1}{3}}$$

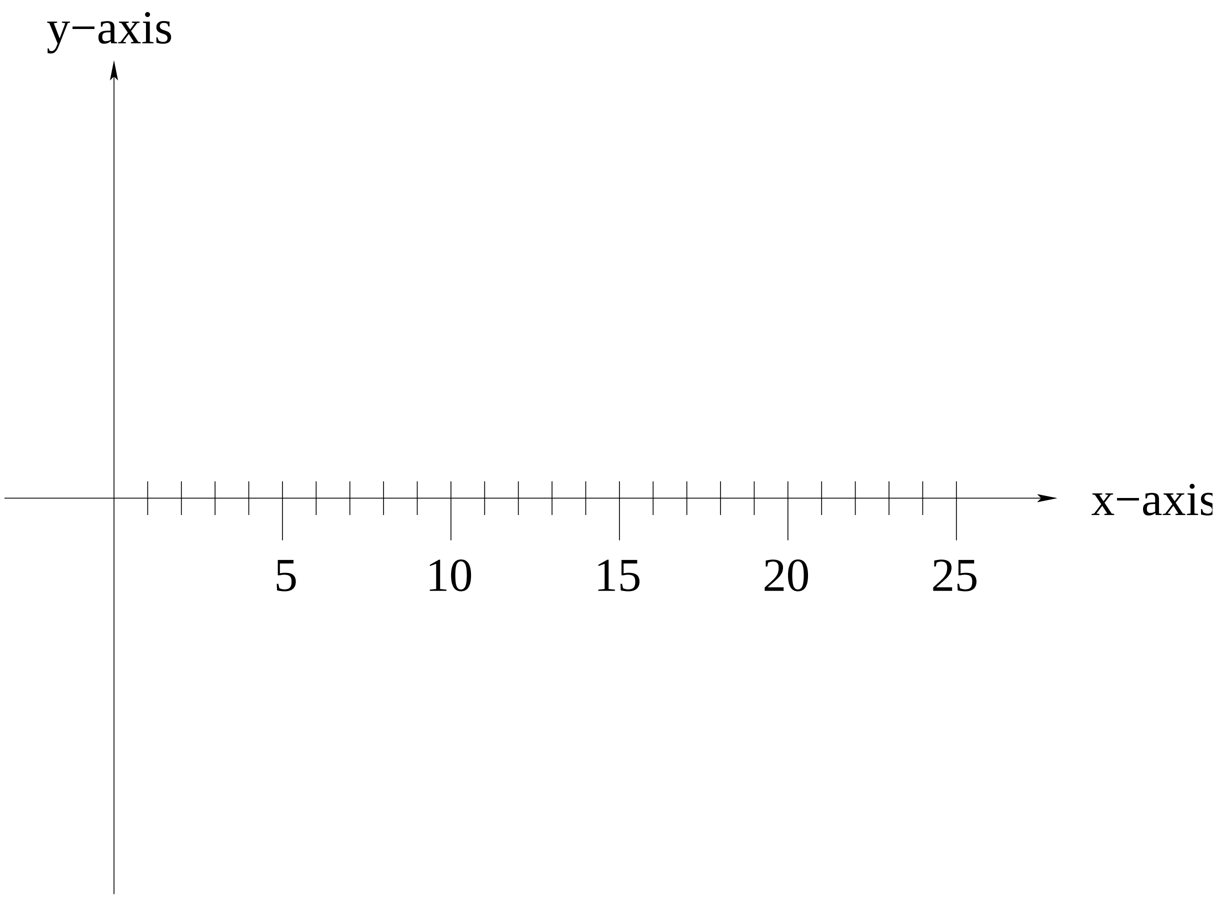
on the domain of all positive numbers.

(a) Find the intervals on which $f(x)$ is increasing, and the intervals on which $f(x)$ is decreasing.

(b) Find the intervals on which $f(x)$ is concave up and concave down.

3. continued.

(c) Sketch the graph of $f(x)$. Label any local extrema, inflection points or asymptotes of the graph.

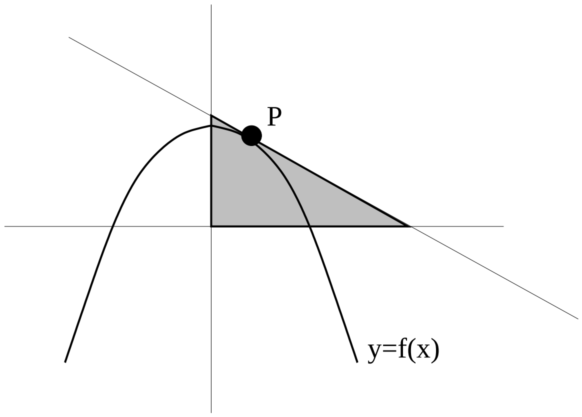


4. (10 points) Compute $\frac{dy}{dx}$ for the following.

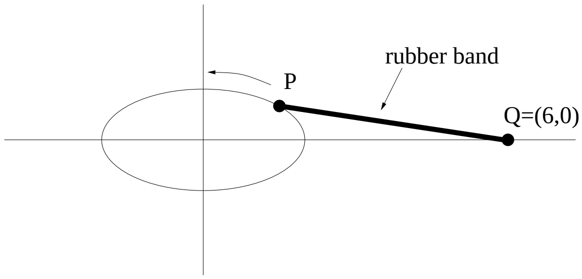
(a) $y = \sin^3\left(\frac{\ln x}{1+x^2}\right)$

(b) $y = e^x \arcsin(\sqrt{x})$

5. (12 points) Consider the graph of $y = f(x) = 1 - x^2$ and a typical point P on the graph in the first quadrant. The tangent line to the graph at P will determine a right triangle in the first quadrant, as pictured below. Find P so that the area of the triangle is as small as possible.



6. (12 points) A rubber band is attached to the locations P and Q in the picture below. The location Q does not change, but the attachment point P moves around the ellipse in a counterclockwise direction according to the parametric equations: $x(t) = 2\cos(\pi t)$ and $y(t) = \sin(\pi t)$, where t is in seconds and all dimensions in the picture are inches.



(a) Find a formula for the length L of the rubber band as a function of time t.

(b) What is the rate of change of the length of the rubber band at $t = 1/2$ second. Leave your answer in exact form.

7. (10 points) Consider the function

$$f(x) = \sqrt{1 + 3\ln(\ln(x) + 1)}.$$

(a) Find the tangent line approximation of $f(x)$ at $x = 1$.

(b) Use the tangent line approximation to estimate $f(1.02)$. Leave your answer in EXACT form.

8. (10 points) Consider the curve given by the equation

$$\tan(x - y) = \frac{y}{1 + x^2}\,.$$

(a) Find **all** the points where this curve intersects the x-axis.

(b) The curve will intersect the x-axis at the point $P = (\pi, 0)$ (one of the answers in part (a).) Using implicit differentiation find the slope of the tangent line to this curve at P.

9. (10 points) Let m and c be positive constants and

$$f(x) = \frac{1}{mx + c}.$$

Using the **limit definition** of the derivative, find the formula for $f'(x)$.