Math 124 Final Examination Autumn 2003

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 10 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	12	
2	12	
3	10	
4	9	
5	9	

Problem	Total Points	Score
6	12	
7	4	
8	10	
9	10	
10	12	
Total	100	

1. (12 points) A robotic car is powered by a solid fuel rocket engine. When the engine is ignited (at $t = 0$), the car accelerates to its top speed and then slows down after the fuel is expended. The car's position along a straight track is given by the formula

$$s(t) = 1 - (1 + rt)e^{-rt}$$

Time is measured in hours (after ignition) and distance in miles. The parameter r is a constant. By varying the rate at which the solid fuel burns, technicians can design engines with the paramter r equal to any positive constant. *Be sure to explain your answers. A one word answer is not good enough*

(a) Find the velocity, $v(t)$, and the acceleration, $a(t)$ as a function of time.

(b) What value of r will result in the vehicle reaching a maximum velocity of exactly 60 miles per hour?

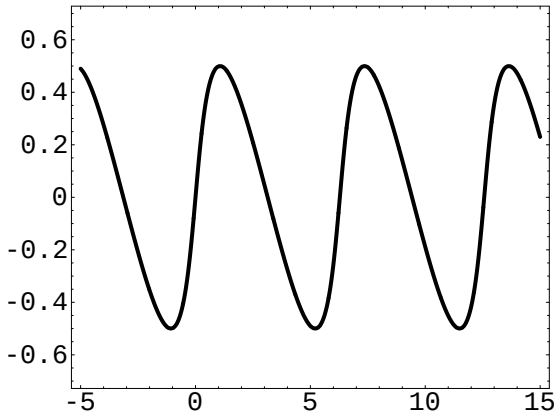
(c) Can they design an engine (choose an $r > 0$) to make the car reach the velocity of sound?

(d) Can they design an engine (choose an $r > 0$) to make the car travel 10 miles away?

2. (12 points) Consider the curve given by the equation

$$2y = \sin(x + y)$$

shown at right.



(a) Compute dy/dx in terms of x and y.

(b) Find the equation of the tangent line to this curve at the point $(\pi, 0)$.

(c) There are three points (x, y) on the curve with $0 \leq x \leq 10$ where the tangent line to the curve is horizontal. Find them. You must give your answers exactly in terms of familiar constants, not as numerical approximations.

3. (10 points) Let $f(t) = t^2 e^{-2t}$ on the domain $D = [-10, 10]$.

   (a) (3pts) Determine the points of D that are critical points of $f(t)$.

   (b) (3pts) Determine the subintervals of D on which $f(t)$ is increasing. Also, determine the subintervals of D on which $f(t)$ is decreasing.

   (c) (3pts) Determine the subintervals of D on which $f(t)$ is concave up. Also, determine the subintervals of D on which $f(t)$ is concave down.

   (d) (1pt) TRUE or FALSE: The absolute maximum value of $f(t)$ on the domain D occurs at an endpoint.

4. (9 points) In this question $f(x)$ and $g(x)$ are unknown differentiable functions, but we know that these functions and their derivatives take the following values at the indicated x-values:

x	0	1	2	3	4
$f(x)$	1	2	3	4	0
$g(x)$	2	3	4	0	1
$f'(x)$	3	2	1	-2	-3
$g'(x)$	2	1	0	0	2

(a) (6pts) Compute the values of the following derivatives. Each answer is worth 2 points; no partial credit.

i. $(fg)'(2)$

ii. $(\frac{f}{g})'(4)$

iii. $(f^3g^4)'(2)$

(b) (3pts) Give a numerical estimate for $(f+2g)(1.01)$. Your answer should be accurate to two decimal places.

5. (9 points) Limit calculations.

(a) Determine

$$\lim_{x \to \pi/2} \frac{\sin x - 1}{\cos x}.$$

(b) Determine

$$\lim_{x \to 1} \frac{4x^3 + 3x - 7}{x^3 - 1}.$$

(c) Determine

$$\lim_{x \to 0} \frac{x + 1}{e^x + 1 + \sin(x)}.$$

6. (12 points) Differentiate the following functions

(a) If $f(t) = (\sin(e^t))^7$, then $f'(t) =$

(b) If $k(t) = \sqrt{1 - t^4}$, then $k'(t) =$

(c) If $m(x) = (x + 1)^{x+2}$, then $m'(x) =$

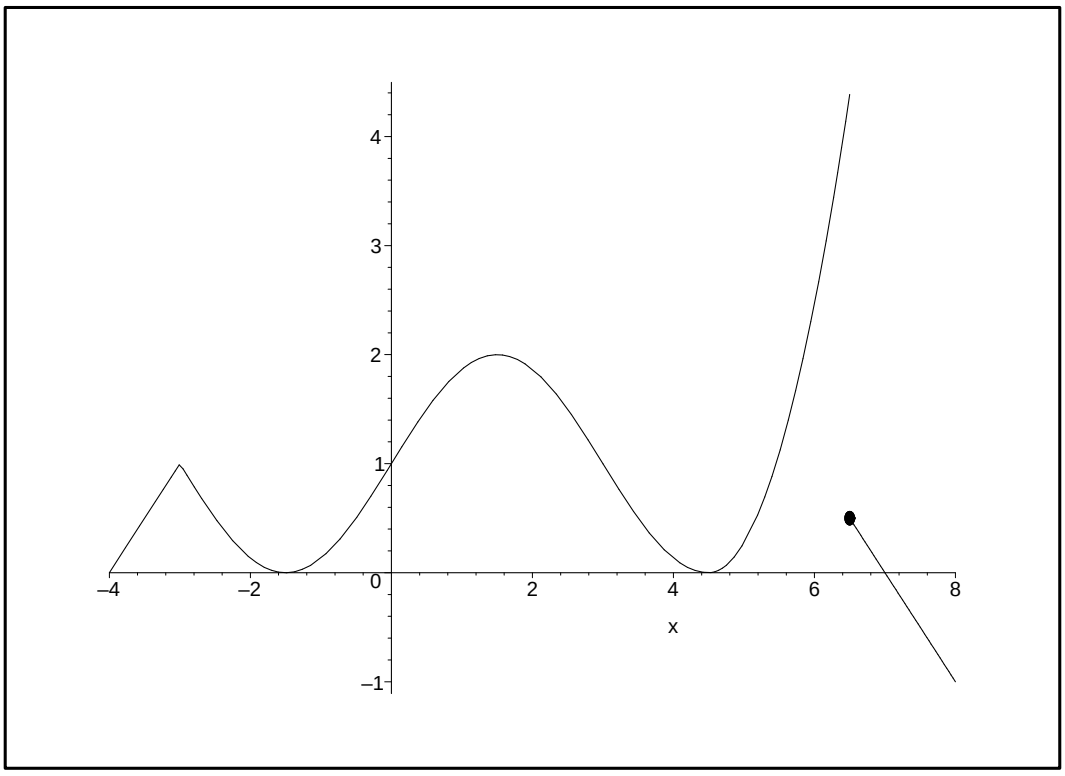
7. (4 points) Compute the derivative of

$$\frac{(1+x)^2(1+x^2)^4(1+x^4)^8}{\sqrt{1+x^2}}$$

8. (10 points) The illumination from a light source is equal to the strength of the source divided by the square of the distance in feet from the source. Assume the light sources are 10 feet apart and have strengths S_1 and S_2. If an object is moving from the first light source to the second light source (on the line segment connecting the light sources) at a rate of 2 inches per second, what is the rate of change of the illumination when the object is midway between the light sources? You should assume that the total illumination from the two sources equals the sum of the illuminations from the specific sources. (Your solution will be an expression involving S_1 and S_2.)

9. (10 points) A piece of wire 80 in long is cut into two pieces. One piece is bent into the shape of a square and the other into the shape of a circle. Where should you cut the wire so that the sum of the areas of the circular and square regions is minimized? Make sure to justify your answer.

10. (12 points) Refer to the graph of the function $f(x)$ below to answer the questions: (No partial credit; each question is worth 2 points.)



(a) Select the correct statement.

i. The function $f'(x)$ is increasing on [1, 2].

ii. The function $f'(x)$ is decreasing on [1, 2].

iii. The function $f'(x)$ is increasing on part of [1, 2] and decreasing on part of [1,2].

iv. None of the above.

(b) Estimate $f'(3)$.

(c) Which statement is true:

i. $f(1.99) < f(2) - 0.01f'(2)$

ii. $f(1.99) = f(2) - 0.01f'(2)$

iii. $f(1.99) > f(2) - 0.01f'(2)$

(d) List all c such that $f'(c)$ does not exist.

(e) Clearly mark and label all inflection points of f in [-2, 6].

(f) How many critical numbers are in the domain $[-3.5, 7.5]$?