Math 124 Final Examination Autumn 2004

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 10 questions and 12 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.

Problem	Total Points	Score
1	9	
2	12	
3	11	
4	8	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (9 points) Differentiate the following functions. You need not simplify your answers.

(a) $y = \dfrac{x^3 + e^x}{\ln(x^2 + 2)}$

(b) $y = x^{2x}$

(c) $y = \arctan(\sqrt{x}) = \tan^{-1}(\sqrt{x})$

2. (12 points) Compute these limits.

(a) $\lim_{x \to 0} \dfrac{x \sin x}{1 - \cos x}$

(b) $\lim_{x \to 0} \dfrac{x \sin x}{1 + \cos x}$

(c) $\lim_{x \to -2^-} \dfrac{|x + 2|}{x^2 + 2x}$

(d) $\lim_{x \to \infty} \dfrac{x - 1}{\sqrt{16x^2 + 5x}}$

3. (11 pts) Answer the questions based on the following table of values.

x	$f(x)$	$f'(x)$	$f''(x)$
0	8	-1	2
1	6	-3	5
2	0	-8	10
3	-2	0	3
4	1	4	-1

(a) (2 pts) $\lim_{x \to 1} \dfrac{f(3-x)^2}{f(3-2x)}$

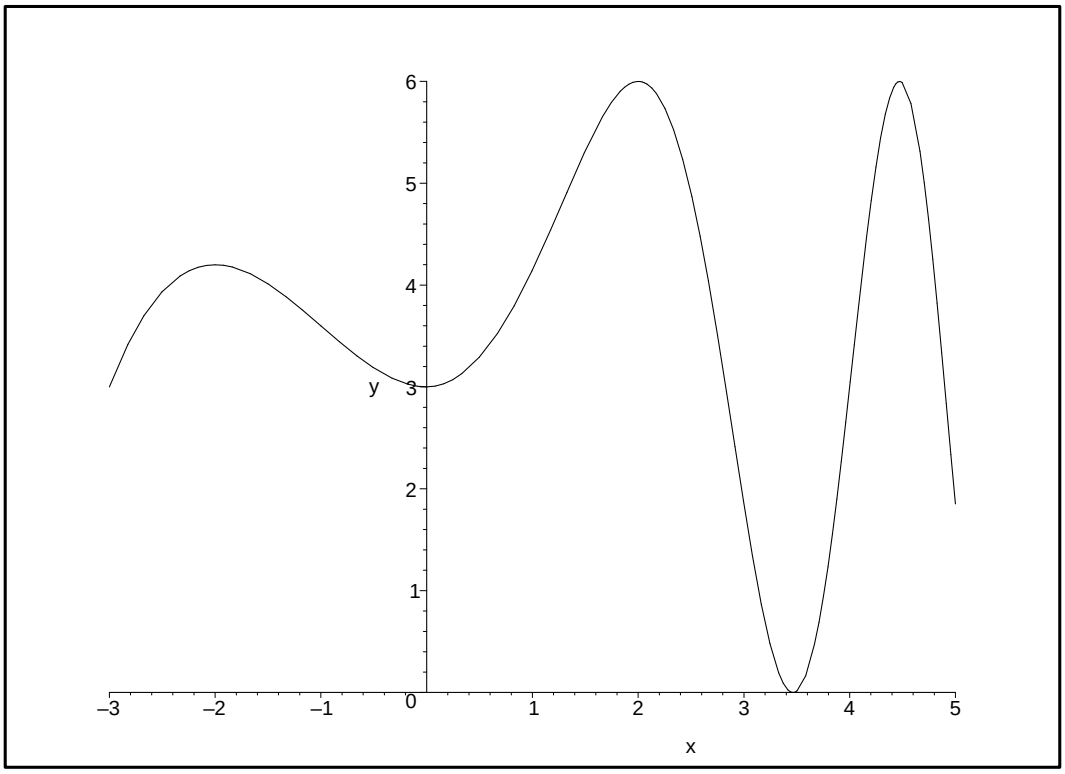
(b) (2 pts) $\lim_{x \to 2} \dfrac{x^2 f(x)}{1 - f(2x)}$

(c) (2 pts) Let $h(x) = (2 + \dfrac{1}{x}) f(x)$. Find $h'(1)$.

(d) (2 pts) Let $r(x) = f(f(x+3))$. Find $r'(-1)$.

(e) (3 pts) Use linear approximation to estimate $f^{-1}(6.1)$.

4. (8 pts) Answer the questions based on the following graph of $y = g(x)$.



(a) (2 pts) Circle the largest.

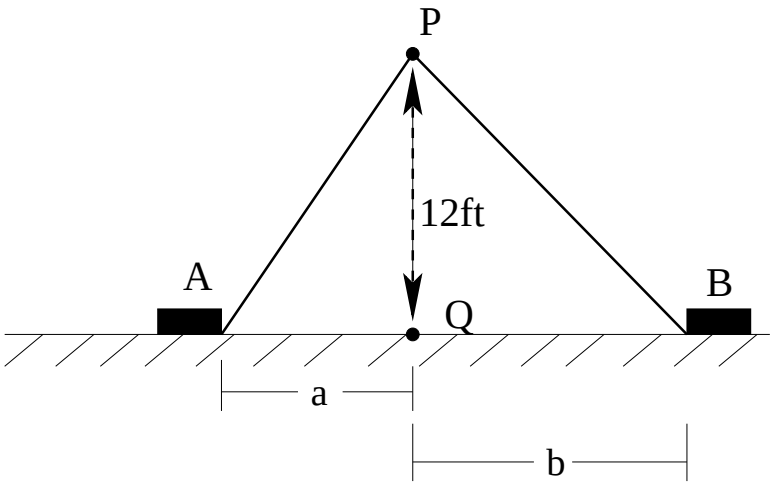
$$g''(-2) \qquad \frac{1}{2}g(3) \qquad g'(1.6) \qquad g'(1.9) \qquad g'(-1)$$

(b) (2 pts) Find a number b such that $g'(b)$ is the absolute minimum for $g'(x)$ in the interval $-3 \leq x \leq 0$.

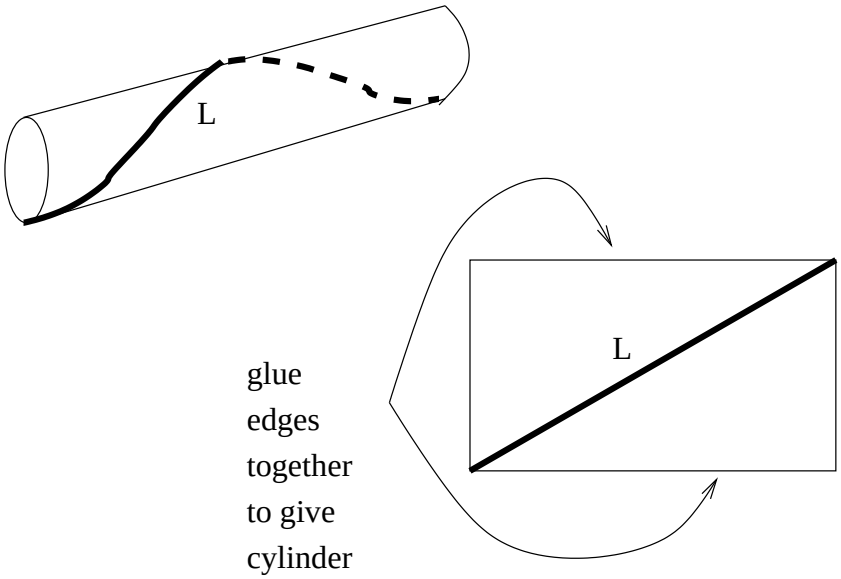
(c) (2 pts) Find a number c such that $g'(c)$ is the absolute maximum for $g'(x)$ in the interval $-1 \leq x \leq 4$.

(d) (2 pts) Find a number d such that $g'(d) = 0$.

5. (10 points) Two carts, A and B, are connected by a rope 33 feet long that passes over a pulley P. The point Q is on the floor 12 ft directly below P and in between the carts. Cart A is being pulled away from Q at a speed of 2 feet/second. How fast is cart B moving toward Q at the instant that cart A is 5 feet away from Q?



6. (10 points) A cylinder is to be made of an elastic material. To support the structure, a wire of fixed length L wraps around the cylinder once as shown below. (A section of vacuum cleaner hose is a good example of this.) Notice, the cylinder can be constructed from a rectangular piece of material as pictured. Varying the dimensions of the rectangle changes the volume of the cylinder. Find the largest volume. Your final answer should be a function of L. (A right circular cylinder has volume $V = \pi r^2 h$.)



7. (10 points) Certain empirical studies model the number of different animal species, N, that exist in a tropical forest with area S, by the equation

$$N = aS^{0.25}$$

where a is a constant. If observations indicate that S is decreasing by 1.8% per year, estimate the yearly percentage decrease in the number of species.

8. (10 points) One point on the curve

$$x^4 + 3xy + y^4 = 5$$

is $(x, y) = (1, 1)$. Use linear approximation to estimate the y value of a nearby point on the curve with $x = 1.1$.

9. (10 points) Here are the parametric equations of a spiral curve:

$$\begin{aligned} x(t) &= t^b \cos(2\pi t) \\ y(t) &= t^b \sin(2\pi t) \end{aligned}$$

For what value of b will the tangent line to the curve through the point $(1, 0)$ be $y = 5x - 5$.

10. (10 points) Let $f(x) = 6x^{1/3} + 3x^{4/3}$ with the domain of all real numbers.

(a) What are the critical points of $f(x)$?

(b) Classify each critical number of $f(x)$ as a local maximum, local minimum or neither.

(c) Determine the subintervals of the domain of $f(x)$ on which $f(x)$ is increasing. Also determine the subintervals on which $f(x)$ is decreasing.

10. continued

(d) Determine the subintervals of the domain of $f(x)$ on which $f(x)$ is concave up. Also determine the subintervals on which $f(x)$ is concave down.

(e) Determine the global extreme points of $f(x)$.