Math 124 **Final Examination** **Autumn 2005**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.
- You have 2 hours and 50 minutes to complete the exam.

Problem	Total Points	Score
1	12	
2	13	
3	8	
4	12	
5	14	
6	13	
7	13	
8	15	
Total	100	

1. [**12 points total**] Differentiate the following functions.

(a) [**4 points**] $g(t) = \sqrt{\dfrac{t^4 - 1}{1 - e^t}}$

(b) [**4 points**] $y = \ln(\arcsin x + \arccos x)$

(c) [**4 points**] $f(x) = x^{e^x \cos(x)}$

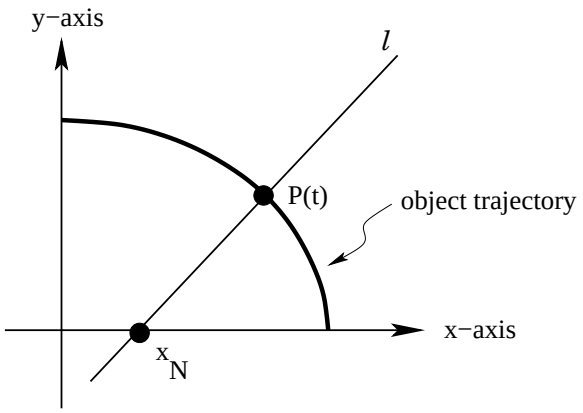
2. [**13 points total**] An object is moving in the xy-plane according to the parametric equations:

$$\begin{aligned} x(t) &= 5\cos(\pi t) \\ y(t) &= 4\sin(\pi t) \end{aligned}$$

When $0 < t < \frac{1}{2}$, the location $P(t) = (x(t), y(t))$ of the object will be in the first quadrant, as pictured below. Let ℓ be the normal line to the trajectory at $P(t)$ and x_N the x-intercept of ℓ.

• The normal line ℓ is perpendicular to the tangent line through $P(t)$.

• Note that the chain rule says that $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$.

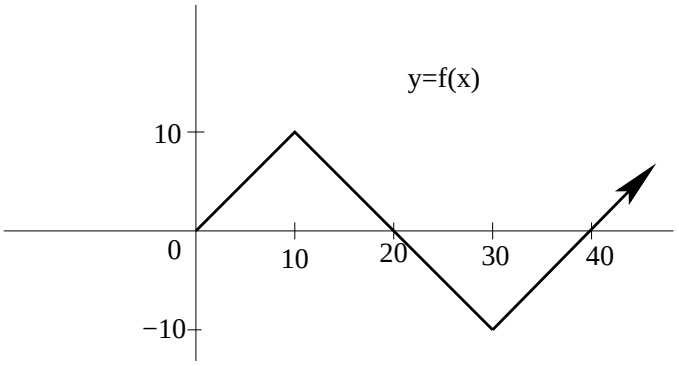


(a) [**6 points**] Write the equation of the normal line ℓ through $P(t)$, assuming $0 < t < \frac{1}{2}$.

(b) [**4 points**] Find an expression for x_N as a function of t.

(c) [**3 points**] Compute $\lim_{t \to 0^+} x_N =$

3. [**8 points total**] Here is the picture of the graph of a distance function $y = f(x)$. Distance is in feet, time is in seconds. Answer these questions; 1 point each, no partial credit.



(a) $f'(7) =$

(b) $\lim_{x \to 0} f(x + 20) =$

(c) $\lim_{x \to 0} \dfrac{f(x + 20)}{x} =$

(d) $\lim_{x \to 0} \dfrac{f(x + 20) - 20}{x - 20} =$

(e) The average velocity on the time interval [0,40] =

(f) The maximum velocity on the time interval [0,40] =

(g) $\lim_{x \to 10} f''(x) =$

(h) Let $g(x) = \dfrac{x}{x + 1}$ and $h(x) = f\left(g(x)\right)$. Find $h'(10) =$

4. [**12 points**] A particle travels in a straight line. After t seconds, the velocity is given by $v(t) = \dfrac{t}{3t+5}$ cm/sec. Compute a formula for the acceleration of the particle at time t.

Use only the limit definition of the derivative and not any differentiation formulas.

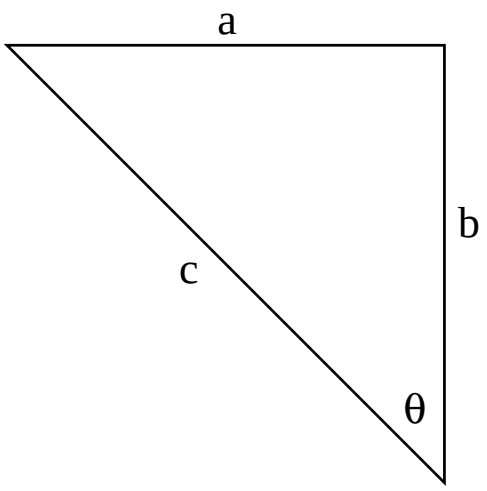
5. [**14 points**] A mass of clay of volume $\frac{4}{3}\pi\,\text{in}^3$ is formed into two spheres. How should the clay be divided to make the total surface area of the two spheres is

(a) a maximum?

(b) a minimum?

Note: the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and its total surface area is $4\pi r^2$.

6. [**13 points**] The hypotenuse of a right triangle is currently 10cm long and increasing at 0.1 cm/sec. The angle θ is $\frac{\pi}{4}$ and decreasing at .001 rad/sec.
How long are the other two sides and how fast are they changing?



7. [**13 points total**] A problem of considerable importance in astronomy, arising in the study of planetary motion, is the determination of the "eccentric anomaly" E of the planet. It satisfies the equation:

$$E - c\sin(E) = \frac{2\pi t}{T}$$

where t is time in years, T is the period of the orbit and c is a constant called the eccentricity. Suppose $c = 0.9$, and $T = 10$ years.

(a) [**4 points**] Find $E'(t)$ when $E = \frac{\pi}{3}$.

(b) [**4 points**] When is $E = \frac{\pi}{3}$?

(c) [**5 points**] Estimate E one month later.

8. [**15 points total**] Consider the function $f(x) = (x-1)\,e^x$.

(a) [**1 point**] What are the x and y-intercepts of the graph of f?

(b) [**3 points**] Using $f'(x)$, calculate the intervals in which f is increasing and decreasing.

(c) [**2 points**] Calculate the minima and maxima of f.

(d) [**3 points**] Using $f''(x)$ calculate the intervals in which f is concave up and concave down.

(e) [**2 points**] Calculate all horizontal asymptotes of f.

(f) [**4 points**] Using all the above information, make a careful sketch of the graph of f, labeling the axes and marking all the above information on the graph.