Math 124 **Final Examination** **Autumn 2006**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	10	
3	12	
4	10	
5	12	
6	12	
7	10	
8	10	
9	12	
Total	100	

1. [**12 points total**] Calculate the derivatives of the following functions. You need not simplfy your answers.

(a) [**4 points**] $y = 5x^7 - \sin(3x) + e^2 + \ln x.$

(b) [**4 points**] $f(r) = \left(1 + \sqrt{r}\right)^3 \cdot \left(1 - \frac{1}{\sqrt{r}}\right)^3$

(c) [**4 points**] $g(t) = (\ln t)^{\ln t}$

2. [**10 points total**] Compute the following limits. You must show your work or adequately explain your answers. No credit will be given for an unsupported answer.

(a) [**5 points**] $\lim_{x \to 2} \frac{\frac{2}{x^2} - \frac{1}{2}}{x - 2}$

(b) [**5 points**] $\lim_{\theta \to 0} \frac{\sin(\theta^2)}{1 - \cos(\theta)}$

3. [**12 points total**] Let $f(x) = e^{2x}\cos(2x)$ on the domain $D = [-1, 2]$.

(a) [**3 points**] Find the subintervals of D on which f is increasing, and the subintervals on which f is decreasing.

(b) [**3 points**] Find the subintervals of D on which f is concave up and concave down.

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(c) [**2 points**] What are the local minima and maxima?

(d) [**2 points**] What are the global minimum and maximum?

(e) [**2 points**] If you used the tangent line approximation to f at $x = 1.0$ to approximate $f(1.1)$, will it give an underestimate or an overestimate. Why?

4. [**10 points total**] Let c be a constant and define $f(x)$ by the piecewise formula

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0; \\ c & \text{if } x = 0. \end{cases}$$

(a) [**2 points**] Find a value of c that makes $f(x)$ continuous on $(-\infty, \infty)$. Use this value of c for the rest of the problem.

(b) [**6 points**] Calculate $f'(0)$. **Use the definition of derivative.** (The differentiation formula won't work here!)

(c) [**2 points**] Is the function $f'(x)$ continuous on $(-\infty, \infty)$?

5. [**12 points total**] Gravitational force always pulls along lines perpendicular to *equi-potential curves.* If two equal masses are located at points, P=(0,4) and Q=(9,0), then the equi-potential curves are described by the condition:

$$\frac{1}{\text{distance from } (x,y) \text{ to P}} + \frac{1}{\text{distance from } (x,y) \text{ to Q}} = \text{Constant.}$$

(a) [**3 points**] Write an implicit equation for the equi-potential curve passing through $(3,8)$.

(b) [**4 points**] What it the slope of the tangent line to this equi-potential curve at $(3,8)$? Please show your work clearly.

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(c) [**3 points**] What is the equation of the line through $(3, 8)$ perpendicular to the equi-potential curve? (This line gives the direction of the gravitational force).

(d) [**2 points**] What angle does the line in part (c) make with the x-axis? (Remember to give units in your answer.)

6. [**12 points total**] A vat contains a salt water solution. Fresh water enters the vat at a constant rate, and the thoroughly mixed solution exits the vat at the same rate. As a result, the concentration of salt is a (decaying) exponential function of time.

Let $y = f(t)$ be the concentration of salt in milligrams per liter at time t hours. Suppose that at time $t = 0$ the concentration is 25.000 mg/ℓ, and two hours later it has been reduced to 20.000 mg/ℓ. Carry out all your work to five significant figures of accuracy.

(a) [**4 points**] Find a formula for y as a function of t. Please show your work clearly.

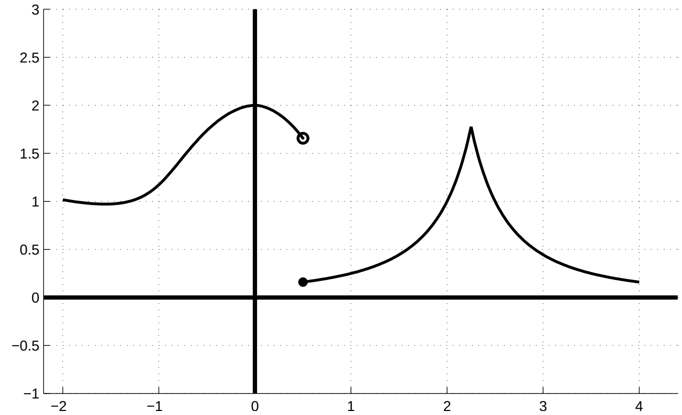
(b) [**4 points**] Find an equation for the tangent line to this function at the point when $t = 2$.

(c) [**4 points**] Using part (b), estimate the concentration of salt after 2 hours 15 minutes. How far is this answer from the exact value obtained using the formula in part (a)?

7. [**10 points total**] A highway patrol officer's radar unit is parked behind a bulletin board 200 ft from a long straight stretch of highway US5. Down the highway, 200 feet from the point on the highway closest to the officer, is an emergency call box. A truck passes the call box and, at that moment, the radar unit indicates that the distance between the officer and the truck is increasing at a rate of 45 miles per hour. The posted speed limit is 55 miles per hour. Calculate the speed of the truck. Should the officer apprehend the driver of the truck for speeding?

8. [**10 points total**] A rectangular poster is to have an area of 2700 cm^2 with 3-cm margins at the bottom and sides and a 5-cm margin at the top. What dimensions will give the largest printed area (the area inside the margins)? Please show your work, and justify that your solution truly is a maximum.

9. [**12 points total**] Consider the function $g(x)$ whose graph is given below. The domain of the function is $[-2, 4]$.



(a) [**2 points**] Is $g(x)$ continuous? Explain briefly why or why not.

(b) [**2 points**] Is the derivative of $g(x)$ continuous? If not, explain where points of discontinuity in the graph of the derivative occur.

(c) [**2 points**] On (approximately) which interval(s) is the derivative positive? Where does the derivative take the value zero?

(d) [**2 points**] On what interval(s) is the *second* derivative negative?

(e) [**4 points**] Sketch a graph of the *derivative* of $g(x)$, making sure to indicate all the features you described in the previous parts.