Math 124 Final Examination Autumn 2008

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 8 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	16	
2	14	
3	10	
4	12	

Problem	Total Points	Score
5	16	
6	10	
7	12	
8	10	
Total	100	

1. (16 points) Find the derivatives of the following functions. You do not have to simplify.

(a) $y = \sqrt{2x^2 + 3x + 5}$, $\qquad \dfrac{dy}{dx} =$

(b) $y = \dfrac{\cos(x)}{x \ln x}$, $\qquad \dfrac{dy}{dx} =$

(c) $y = (\tan x)^{x^2}$, $\qquad \dfrac{dy}{dx} =$

(d) $y = \sin^{-1}(e^{3x})$, $\qquad \dfrac{dy}{dx} =$

2. (14 points) For this problem, consider the function $y = f(x) = \frac{1}{4}\cos(2\pi x) + \frac{\pi x}{4}$ for $0 \leq x \leq 1$. Please show your work, and leave your answers in exact form (not decimal form).

(a) Find all of the critical numbers c for $f(x)$.

(b) Use the second derivative test to determine in each case whether there is a local maximum or local minimum at c.

(c) Find the x and y coordinates of all of the inflection points (if any).

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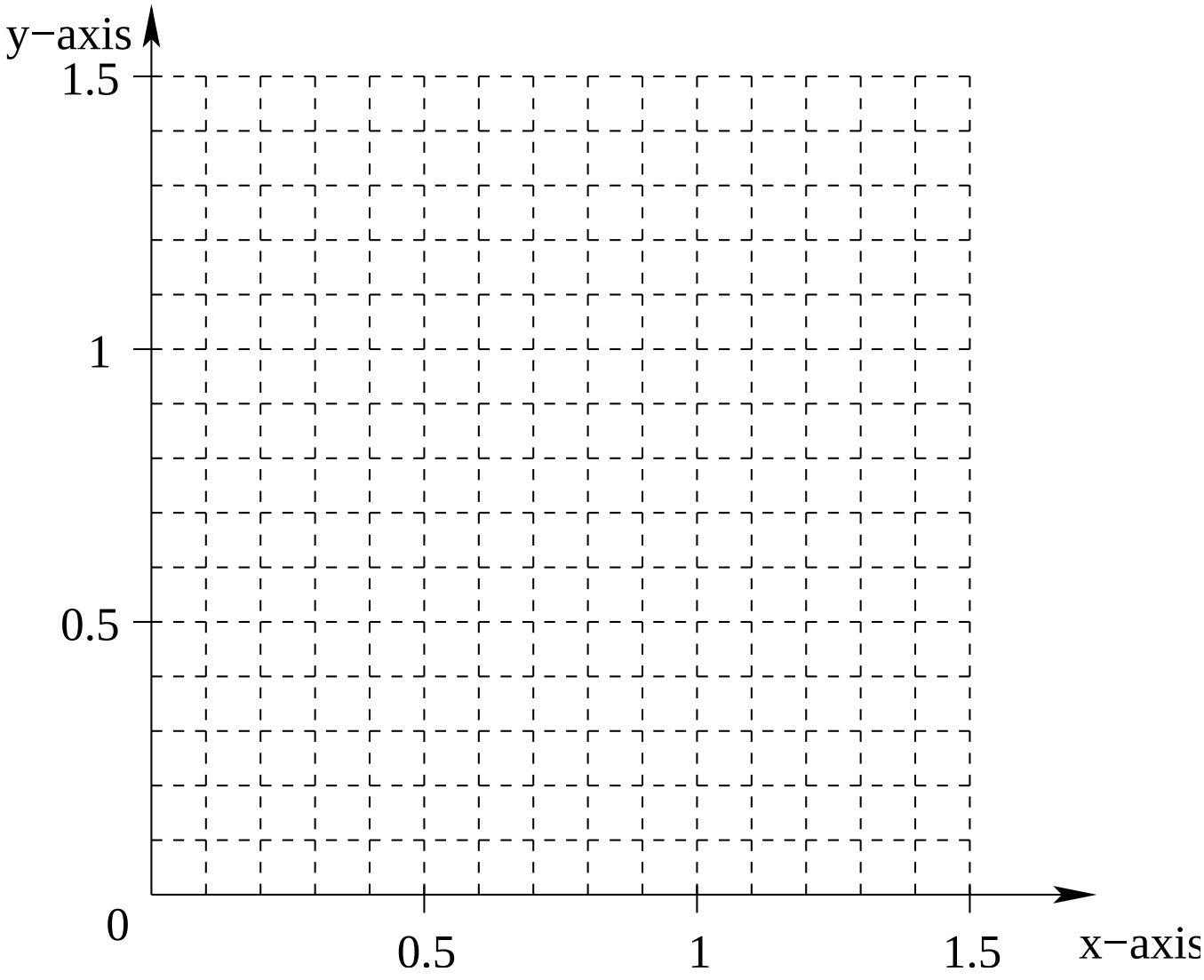
2. continued.

(d) Sketch the curve. Make sure to label any points on the graph corresponding to local maximum or local minimum values of the function. Also, label any inflection points.



3. (10 points) A particle is traveling along a curve. At time t, its position is given by

$$\begin{aligned} x(t) &= t^3 + t^2 + t \\ y(t) &= t^4 + t^3 + t^2 + 1. \end{aligned}$$

(a) Find the equation of the tangent line to the curve at time $t = 1$.

(b) Shortly after the time $t = 1$, the particle is at the point with x-coordinate 3.1. Use the tangent line approximation to estimate its y-coordinate at that time.

4. (12 points) A plastic cup is made in the shape of a cylinder (without a top). The volume of the cup is 250 cm^3. Find the dimensions (radius and height) of the cup which minimize the amount of plastic required. (The thickness of the cup is fixed and is small in comparison to the other dimensions.)

5. (16 points) Find the following limits or explain why the limit does not exist. Give exact answers.

(a) $\lim_{x \to 0} \dfrac{\sin x}{|x|} =$

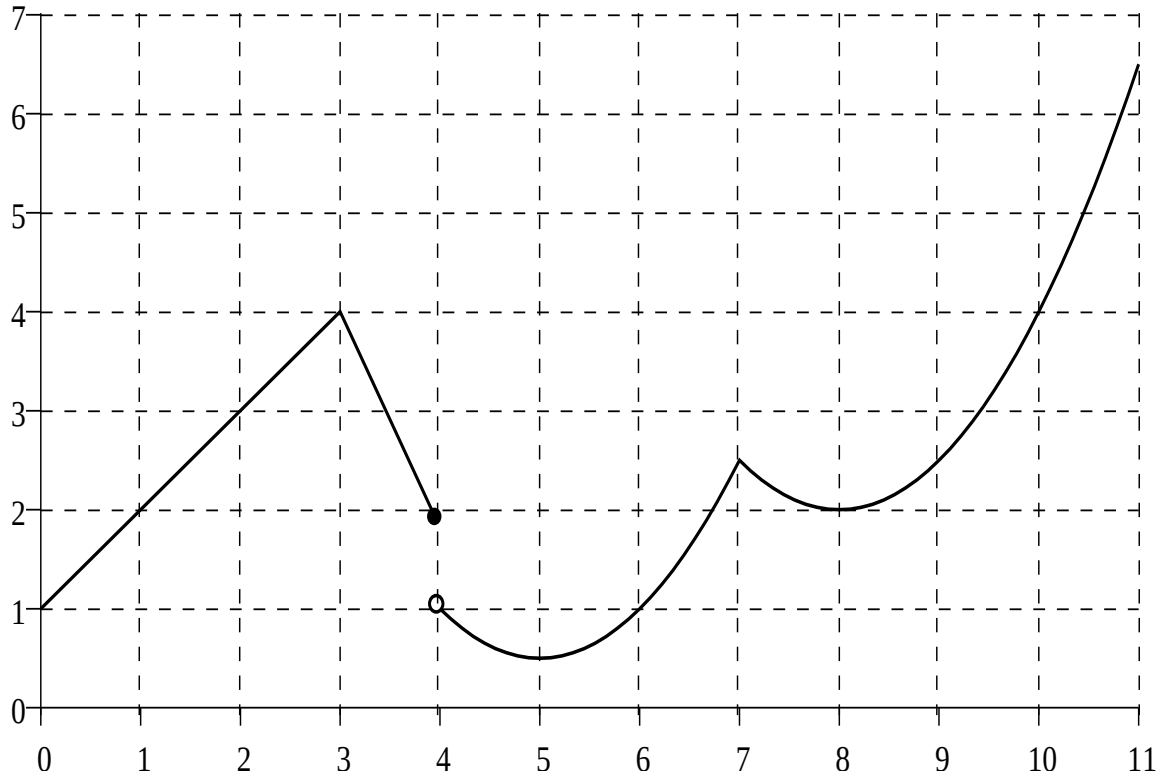
(b) $\lim_{x \to 3} \dfrac{e^{x-3} - 1}{x^2 - 4x + 3} =$

(c) $\lim_{x \to \infty} \sin^{-1} \left(\dfrac{x(x^4 + 1)}{2x^5 - 3x^2 + x + 7} \right) =$

(d) $\lim_{x \to 0} \dfrac{\sin x + \cos x}{x - \cos x} =$

6. (10 points) A particle moves in the xy-plane so that its x- and y-coordinates are differentiable functions of time. When the particle is located at $(x, y) = (-8, 6)$, $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = -2$. At that instant, what is the rate of change of the distance from the particle to the origin?

7. (12 points) Below is the graph of the function $g(x)$. The domain of $g(x)$ is the interval $0 \leq x \leq 11$. Use the graph to answer the following questions. No justification is required.



(a) Is $g(x)$ continuous everywhere? If not, give all x-values where $g(x)$ is not continuous.

(b) Is $g(x)$ differentiable everywhere? If not, give all x-values where $g(x)$ is not differentiable.

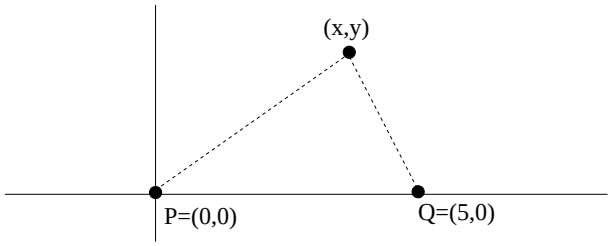
(c) Find all values x where $g'(x) = 0$.

(d) Find the minimum value of $g'(x)$.

(e) Compute $g''(2)$

(f) Compute $\lim_{h \to 0^+} \dfrac{g(3+h) - g(3)}{h}$

8. (10 points) In a two-dimensional universe the *gravitational potential* is proportional to the natural logarithm of the distance from the source. Suppose that your two-dimensional universe contains two stars — a large one P at the origin and a smaller one Q at the point $(5, 0)$ on the x-axis. If you are standing at a general location (x, y) in this two-dimensional universe, the total gravitational potential will depend on both P and Q:



Assume star P has 5 times the mass of star Q, you're at the specific point $(4, 3)$ and want to move so that the total gravitational potential from the two stars remains constant. Then this means we have the equation:

$$5\ln(\text{distance from P}) + \ln(\text{distance from Q}) = \text{C},$$

where C is a constant. This path is called the "equipotential" curve.

(a) Use the fact that the equipotential curve passes through the point $(4, 3)$ to find the constant C on the right. Then write out an implicit equation for the equipotential curve passing through $(4, 3)$.

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8. continued

(b) Find the slope of the tangent line to the equipotential curve at the point $(4, 3)$.