Math 124 Final Examination Spring 2004

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 10 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.

Problem	Total Points	Score
1	12	
2	12	
3	14	
4	12	
5	10	

Problem	Total Points	Score
6	8	
7	8	
8	12	
9	12	
Total	100	

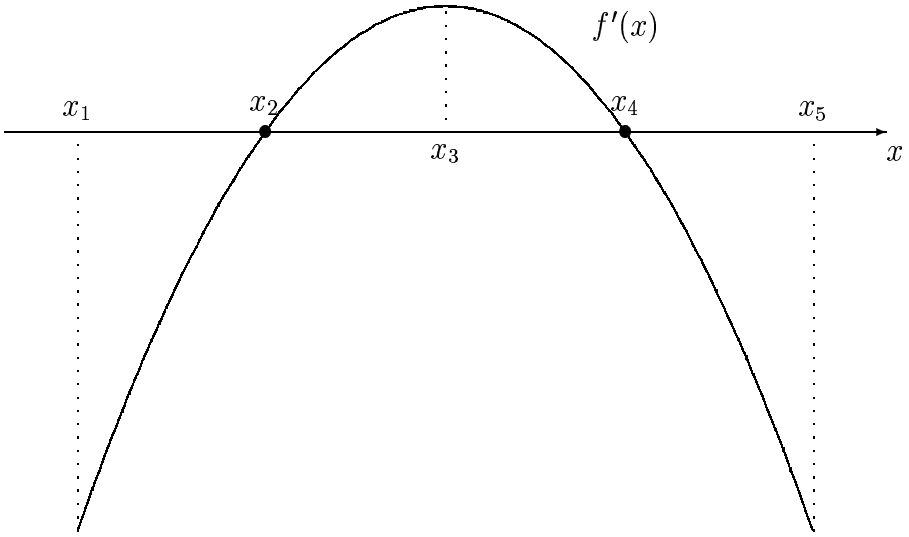
1. (12 points) Differentiate the following functions. You need not simplify your answers.

(a) $f(x) = \left(\dfrac{x^2+1}{x^4+2}\right)^{50}$

(b) $g(x) = \left(\dfrac{2}{x}\right)^{1/x}$

(c) $h(x) = \cos^2(\sin(x))$

2. (12 points) Below is the graph of $f'(x)$.



(a) Give all values of x at which:

i. $f(x)$ has a horizontal tangent;

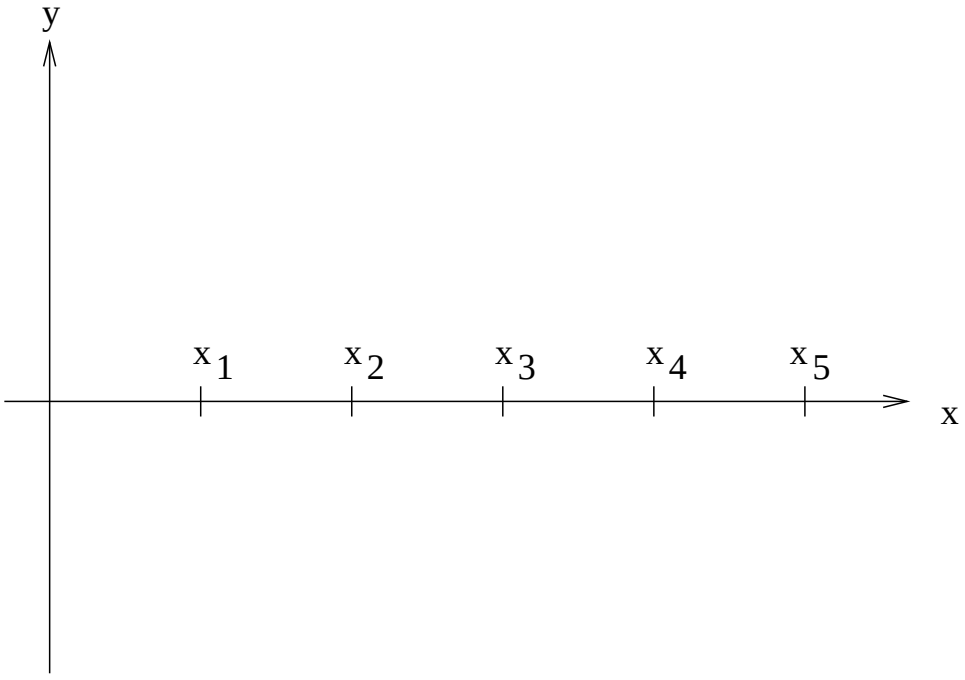
ii. $f(x)$ is increasing;

iii. $f(x)$ is concave down;

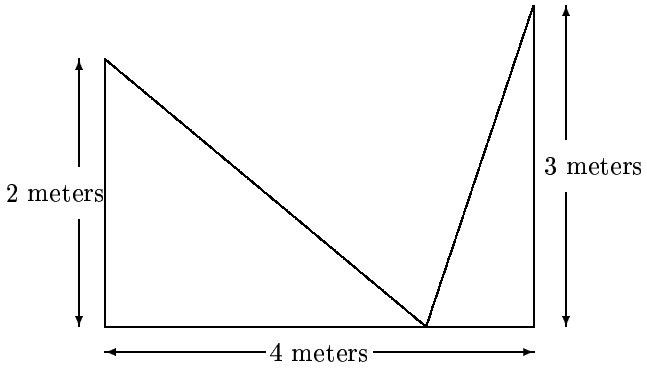
iv. $f''(x) = 0$;

v. $f(x)$ has a local minimum.

(b) Sketch a graph that could represent $y = f(x)$, assuming that $f(x_1) = 0$.



3. (14 points) A 2-meter vertical pole is located 4-meters from a 3-meter vertical pole as pictured below. The poles are to be reinforced by a wire connected to the top of the poles and tied to a stake driven into the ground between them. How far from the shorter pole should the stake be driven in order to minimize the length of the wire?



4. (12 points) If a projectile is fired with an initial velocity of v_0 meters per second at an angle α above the horizontal, then its position after t seconds is given by the parametric equations:

$$x = x(t) = (v_0 \cos \alpha)t$$
$$y = y(t) = (v_0 \sin \alpha)t - 5t^2.$$

A gun is fired from the ground with $\alpha = \frac{\pi}{6}$ radians and the bullet hits the ground 20 seconds later.

(a) What is the initial velocity of the bullet?

(b) How far from the gun does the bullet hit the ground?

(c) Eliminate the parameter t to find a formula for y in terms of x.

(d) Find the equation of the line tangent to the bullet's path at $t = 12$ seconds.

5. (10 points) A rocket is launched vertically from a point on the ground that is 100 horizontal meters from an observer with binoculars. The rocket is rising vertically and its height above the ground (in meters) is given by

$$y(t) = 60t - 5t^2$$

Two seconds after the launch, how fast must the observer change the angle of elevation of her line of sight to keep the rocket in sight?

6. (8 points) Find the equation of the tangent line to the curve given by

$$x^3 - y^3 = 2xy$$

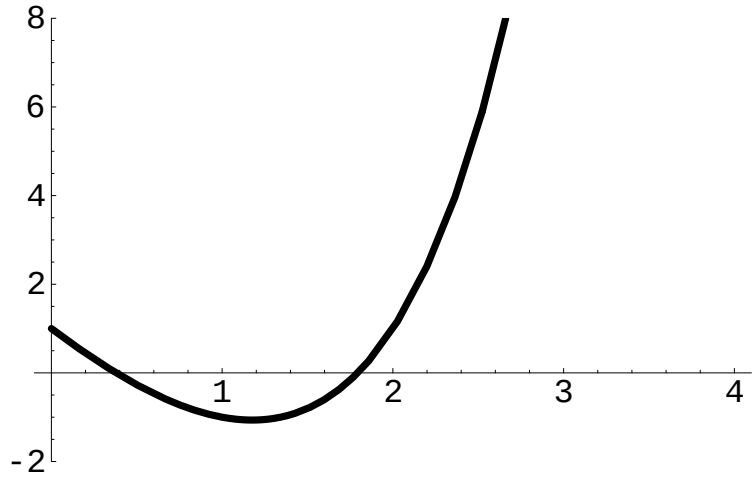
at the point $(-1, 1)$.

7. (8 points) Find values of a and b so that the line $2x + 3y = a$ is tangent to the graph of $f(x) = bx^2$ at the point where $x = 3$.

8. (12 points)

(a) Find the tangent line approximation (i.e. linear approximation) of the function $y = f(x) = 3^x - 4x$ at $x = 2$.

(b) The equation $3^x - 4x = 0$ has two solutions: one is between 0 and 1 and the other is a little less than 2. The graph of $f(x) = 3^x - 4x$ is given below:



Use linear approximation (i.e. the tangent line approximation) to estimate the value of the solution near 2. Give your answer accurate to two decimal places.

9. (12 points) Evaluate the following limits and justify your answers. Give exact answers to each of (a)-(c). (Plugging values into your calculator is not a justification.)

(a) Compute $\lim_{x \to 0} \left(\dfrac{\sin(x) - x}{x^3} \right)$

(b) Let $H(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u \leq 0 \end{cases}$

Compute $\lim_{x \to 0} H(x^2)$

(c) Compute $\lim_{x \to \infty} (e^{-x} + e^{-2x})^{1/x}$