

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 12 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, 3π , $\sqrt{2}$, $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, 0.693147 are NOT in exact form.

Problem	Total Points	Score
1	15	
2	12	
3	10	
4	9	
5	12	

Problem	Total Points	Score
6	12	
7	10	
8	8	
9	12	
Total	100	

1. (15 points) Make sure to justify your answers in each part below. Let

$$f(x) = \begin{cases} \frac{1 - \cos(x^2)}{x^4} & \text{if } x < 0 \\ \frac{3x^2 + 2}{4x^2 + 4} & \text{if } x > 0 \end{cases}$$

(a) (3 pts) Find $\lim_{x \rightarrow 0^-} f(x)$.

(b) (3 pts) Find $\lim_{x \rightarrow 0^+} f(x)$.

(c) (3 pts) Suppose $f(0) = c$, where c is some number that you may choose. Can you choose a value of c to make f continuous at $x = 0$? If so, give that value of c ; if not, explain why not.

(d) (3 pts) Find $\lim_{x \rightarrow -\infty} f(x)$.

(e) (3 pts) Find $\lim_{x \rightarrow +\infty} f(x)$.

2. (12 points) Define the function $y = f(x)$ on the domain $D = [0, 1]$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ x \ln(3x) + 1 & \text{if } 0 < x \leq 1 \end{cases}$$

(a) (3 pts) Determine the critical points of $f(x)$.

(b) (3 pts) Determine the subintervals of D on which $f(x)$ is increasing. Also determine the subintervals of D on which $f(x)$ is decreasing.

2. (continued) Recall the function $y = f(x)$ on the domain $D = [0, 1]$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ x \ln(3x) + 1 & \text{if } 0 < x \leq 1 \end{cases}$$

(c) (3 pts) Determine the subintervals of D on which $f(x)$ is concave up. Also, determine the subintervals of D on which $f(x)$ is concave down.

(d) (3 pts) Determine the absolute maximum and absolute minimum values of $f(x)$ on the domain D , and find all points x in D where $f(x)$ takes on these values.

3. (10 points) The location of a particle in the xy -plane at time t seconds is defined by the parametric equations

$$\begin{aligned}x &= \sin t \\y &= e^{t-\pi} \cos t\end{aligned}$$

- (a) (2 pts) Find the derivative $\frac{dx}{dt}$ as a function of t .

- (b) (2 pts) Find the derivative $\frac{dy}{dt}$ as a function of t .

- (c) (2 pts) Find the derivative $\frac{dy}{dx}$ as a function of t .

- (d) (4 pts) Find the equation of the tangent line to the curve at the point where the particle is at time $t = \pi$ seconds. Give your answer in the form “ $y = mx + b$ ”.

4. (9 points) A balloon is at a height of 40 meters, and is rising at the constant rate of 8 m/sec. At that instant a bicycle passes beneath it, traveling in a straight line at the constant speed of 10 m/sec. How fast is the distance between them increasing 3 seconds later?

5. (12 points) Two factories are located 8 miles apart at points A and B on a straight road. Both emit smoke, but the amount of pollutant coming from B is three times that coming from A , as measured by particulate count. The intensity of pollutant from a factory is equal to the amount of pollutant coming from that factory divided by the distance in miles from the factory, and the total intensity at a point is the sum of the intensities due to the two factories. A contractor wants to build a house along the stretch of road connecting the two factories at a point where the total intensity is minimal. Where should the contractor put the house?

6. (12 points) Compute the following derivatives. Make sure to show your work, but DO NOT SIMPLIFY:

(a) (3 pts) If $h(x) = \cos(\tan(2x))$, find $h'(x)$.

(b) (3 pts) If $g(x) = (ax^2 + b)e^{-cx}$, find $g'(x)$. Assume a, b, c are constants.

6. (continued)

(c) (3 pts) If $y = \frac{3x \ln(x)}{2x^3 - x + 7}$, find y' .

(d) (3 pts) If $y = (\sin(x))^{x^2}$, find $\frac{dy}{dx}$.

7. (10 points) Consider the curve defined by the equation $y^3 + x^2y - 2x^3 = 8$.
- (a) (5 pts) Find the equation of the tangent line to the curve at the point $(1, 2)$. Give your answer in the form " $y = mx + b$ ".
- (b) (5 pts) Find the x and y coordinates for all points where the tangent line to the curve is horizontal.

8. (8 points) Questions (a) and (b) refer to these graphs:

(a) (4 pts) Given the graph of $y = f'(x)$ below, select a graph above which best represents the graph of $y = f(x)$.

ANSWER _____
(Choose one of A-H.)

(b) (4 pts) Given the graph of $y = g(x)$ below, select a graph above which best represents the graph of $y = g'(x)$.

ANSWER _____
(Choose one of A-H.)

9. (12 points) A pizza delivery chain has found that the number of veggy pizzas it can sell on a Friday night is closely modeled by the function

$$N(x) = 1000 - 240 \ln(x/12) - 20x,$$

where x is the cost of the pizza in dollars. The current price is \$12. The pizza chain want to have their sales on Friday night increase by 50 pizzas. Use linear approximation to estimate the required new pizza price.