Math 124 Final Examination Winter 2005

Print Your Name	Signature
Student ID Number	**Quiz Section**
Professor's Name	**TA's Name**

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 8 questions and 10 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.

Problem	Total Points	Score
1	12	
2	12	
3	15	
4	15	
5	10	

Problem	Total Points	Score
6	12	
7	12	
8	12	
Total	100	

1. (12 points; 4 points each) Differentiate the following functions. You need not simplify your answers.

   (a) $y = (\sin(x))^x$

   (b) $y = \ln\left(\left(\frac{x^2+1}{x^3+2}\right)^{100}\right)$

1. (continued)

(c) $y = \sin\left(\sqrt[3]{\frac{t^2}{7} + 5}\right)$

2. (12 points; 4 points each) Compute these limits.

(a) $\lim_{x \to \infty} \frac{x}{\ln(1 + 2e^x)}$

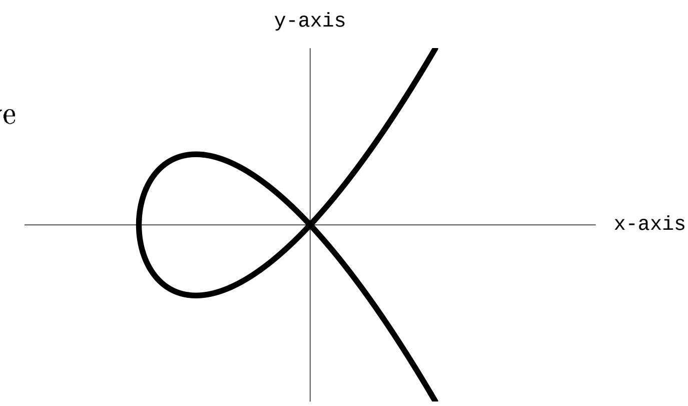
(b) $\lim_{x \to 0} \frac{(2 + x)^2 - 2(2 + x)}{x}$

(c) $\lim_{t \to 0^+} t \ln(t^2)$

3. (15 points; 5 points each part) The curve with equation

$$y^2 = x^3 + 3x^2$$

has a graph as pictured:



(a) Find an equation of the tangent line to this curve at the point $(1, -2)$.

(b) Let $P = (b, -2.1)$ be the point on the curve whose y-coordinate is -2.1. Using linear approximation and part (a), estimate b.

(c) Find the coordinates of all points where the tangent line to the curve is horizontal.

4. (15 points; 5 points each part) Let

$$f(x) = 1 - e^{-x}\cos x$$

(a) Find the x-coordinate of all *critical points* of $f(x)$ in $(0, 2\pi)$.

(b) Determine if the critical points you found are local maxima, local minima or points of inflection.

(c) Locate the *global minimum* of $f(x)$ in the interval $[0, 2\pi]$.

5. (10 points) A particle is moving in the plane according to the parametric equations

$$\begin{aligned} x &= \cos t + 2t \\ y &= \sin t; \end{aligned}$$

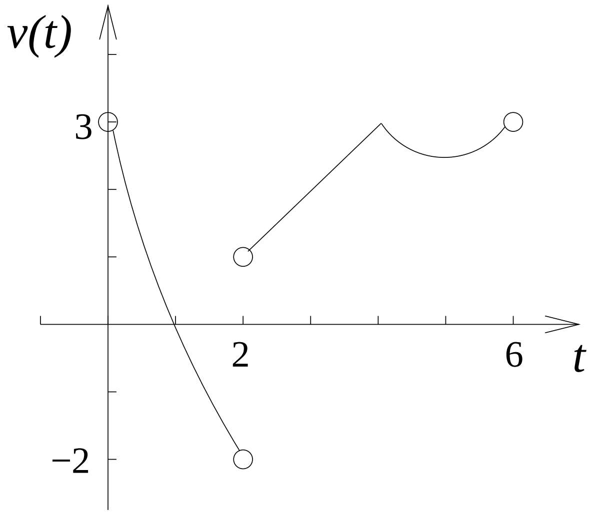
t is time and $t \geq 0$.

(a) Find the velocities $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(b) Using the chain rule, find $\frac{dy}{dx}$ at all times t when $y = 0$.

6. (12 points) You are on a ship heading North at 12 miles per hour toward an island when you spot a sailboat heading East away from the same island. When your ship is 9 miles from the island, the angle measured clockwise from due North to the boat is seen to be $\frac{\pi}{6}$ radians and the angle is increasing at $\frac{\pi}{3}$ radian/hour. How fast is the sailboat moving?

7. (12 points) A charged particle moves along the x-axis in the presence of a fluctuating force field. Let $x(t)$ be the x-coordinate of the particle at time t. The graph of the particle's velocity $v(t) = x'(t)$ in the positive x-direction is shown.



○ means the point is not in the graph

(Due to sudden changes in the force field, the velocity is sometimes discontinuous.) Answer the following questions, worth 2 points each. NO PARTIAL CREDIT.

(a) Find $\lim_{t\to 2} v(t)$.

(b) Find $\lim_{t\to 4} v(t)$.

(c) Find $\lim_{t\to 1^-} v(2t)$.

(d) Find $\lim_{t\to 1} \dfrac{x(t)-x(1)}{t-1}$.

(e) Find a time t when the particle has zero acceleration.

(f) On the interval $(1, 2)$, is the particle speeding up or slowing down?

8. (12 points) Jane is located 8 km out from the nearest point A along a straight shoreline in her sea kayak. Hunger strikes and she wants to make it home for lunch; see picture. Jane can paddle at 3 km/hr and walk at 6 km/hr. If she paddles along a straight line course to the shore, find an equation that computes the the total time to reach lunch in terms of the location where Jane beaches the kayak. Where should she beach the kayak to eat as soon as possible?