Math 124 Final Examination Winter 2008

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 8 questions and 12 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	

Problem	Total Points	Score
5	12	
6	16	
7	12	
8	12	
Total	100	

1. (12 points) Compute the following limits. You must show your work or adequately explain your answers. No credit will be given for an unsupported answer.

   (a) (6pts) $\lim_{x \to 1^+} \dfrac{x^2 + 4x - 5}{x^2 - 2x + 1}$

   (b) (6pts) $\lim_{x \to 0} \dfrac{1 - \cos(x)}{\sin(x^2)}$

2. (12 points) Compute $\dfrac{dy}{dx}$ for the following.

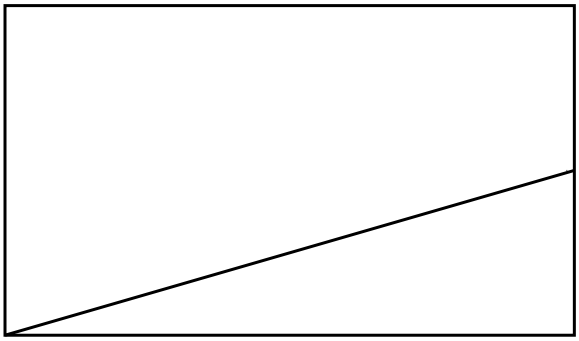
(a) (4pts) $y = \dfrac{3x^2 + \ln x}{\sin(e^x)}$

(b) (3pts) $y = x \arctan(\sqrt{x})$

2. continued

(c) (5pts) $y = x^{\sin(x)}$

3. (12 points) A rectangle has area 2 in^2. What is the minimum length of a line drawn from one corner to the midpoint of one of the two more distant sides? Please show your work, and justify that your solution truly is a minimum.



4. (12 points) An object moves following the law of motion

$$s = s(t) = \frac{t^2}{8} + \ln(1+t)$$

where $t \geq 0$ is measured in seconds and s in meters.

(a) (3pts) Find the velocity at time t.

(b) (3pts) When is the velocity positive?

(c) (3pts) Find the acceleration at time t.

(d) (3pts) Find the time $t \geq 0$ at which the velocity is minimal.

5. (12 points) Tank A has the shape of a cube which is 30 feet on a side. It is full of water. Tank B has the shape of a cylinder with circular base of radius 20 feet. It is empty. The water is to be pumped from from Tank A into Tank B. At a certain moment, the level of the water in Tank A is dropping by 3 inches/minute. How fast is the water rising in Tank B?

6. (16 points) Let $f(x) = x^4 e^{-x}$.

(a) (4pts) Find the intervals on which $f(x)$ is increasing, and the intervals on which $f(x)$ is decreasing.

(b) (2pts) What are the local minima and maxima?

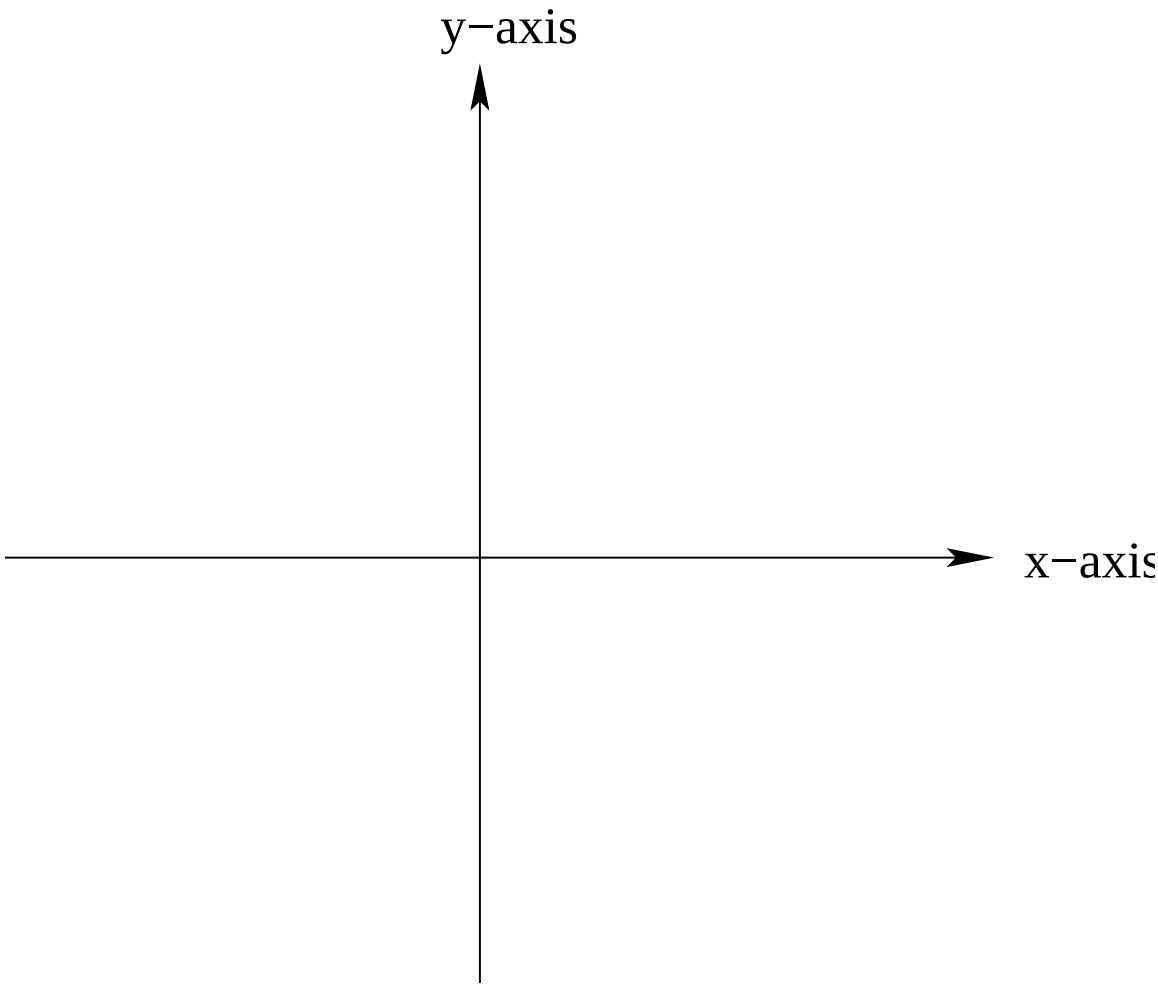
6. continued

(c) (4pts) Find the intervals on which $f(x)$ is concave up and concave down.

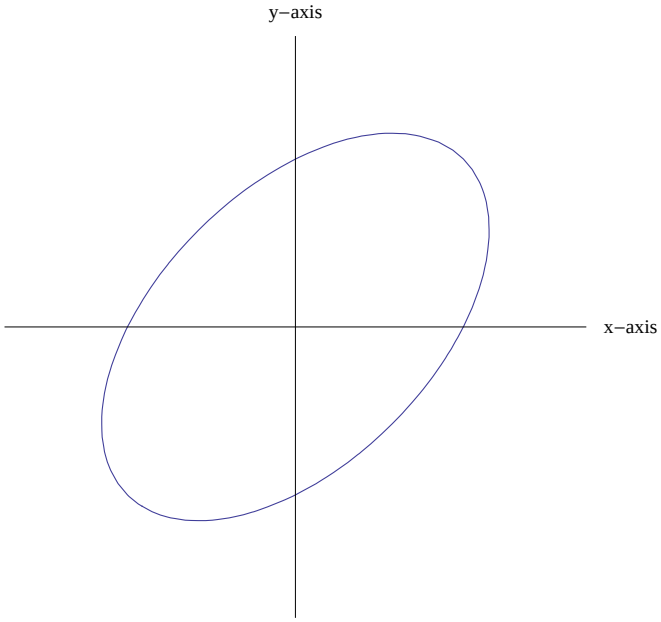
(d) (2pts) Is there a global minimum or maximum?

6. continued

(e) (4pts) Use the curve sketching procedure to carefully and clearly graph $f(x)$. (Include BOTH coordinates of all local minima and maxima, inflection points, x-intercept(s) and y-intercept(s).)



7. (12 points) Find the largest y coordinate of the points on the ellipse $x^2 - xy + y^2 = 3$.



8. (12 points) Use linearization at $(0,0)$ to estimate the y value of a point $(0.002, y)$ on the curve $3y = \arctan(y - x)$ near $(0,0)$.