Math 124 Final Examination Winter 2009

Print Your Name | Signature

Student ID Number | Quiz Section

Professor's Name | TA's Name

!!! READ...INSTRUCTIONS...READ !!!

1. Your exam contains 9 questions and 12 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. BOX IN YOUR FINAL ANSWER FOR EACH QUESTION.
6. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
7. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example, $3\pi, \sqrt{2}, \ln(2)$ are in exact form; the corresponding approximations $9.424778, 1.4142, 0.693147$ are NOT in exact form.

Problem	Total Points	Score
1	12	
2	10	
3	12	
4	8	

Problem	Total Points	Score
5	12	
6	10	
7	12	
8	10	
9	14	
Total	100	

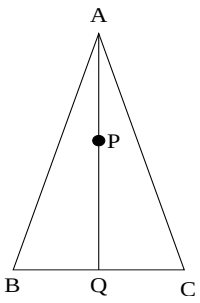
1. (12 points) Find the derivatives of the following functions. You do not have to simplify.

(a) $y = \dfrac{x}{e^{\pi x^2}}$, $\qquad \dfrac{dy}{dx} =$

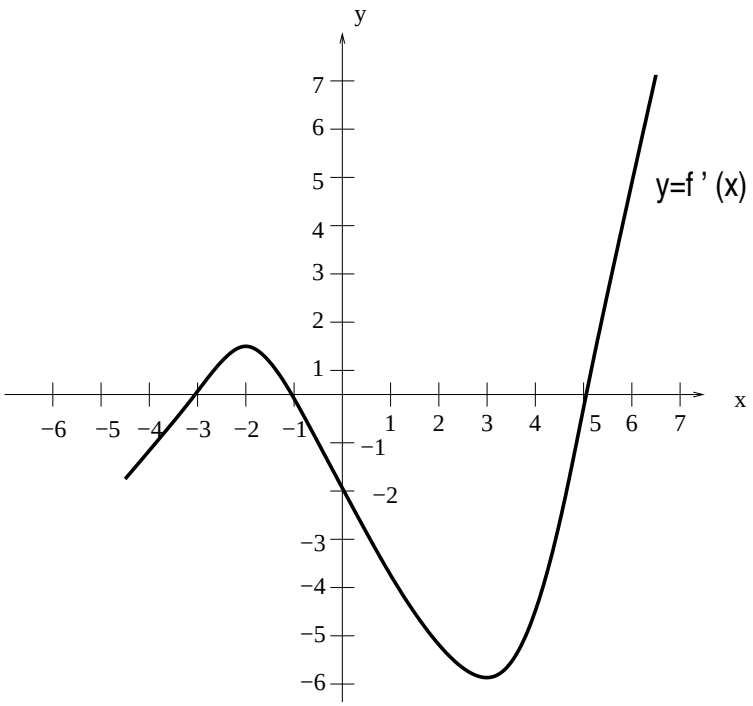
(b) $y = \ln\left(x^2 - \tan x\right)$, $\qquad \dfrac{dy}{dx} =$

(c) $y = (1 + \cos x)^{\sin x}$, $\qquad \dfrac{dy}{dx} =$

2. (10 points) The isosceles triangle shown below has height AQ of length 3 and base BC of length 2. A point P is placed along the line segment AQ. What is the minimum of the sum of the distances from P to A, P to B, and P to C? (Make sure to justify your final answer.)



3. (12 points) Assume that f is a differentiable function with $f(0) = 0$. **The graph of its derivative function f' is shown below.** No partial credit on this problem; you need not justify your answers. BOX in the answer you want graded in each part.



(a) On what interval(s) is f increasing?

(b) On what interval(s) is f decreasing?

(c) Is $f(-1)$ positive, negative, or 0?

(d) For what values of x does f attain a local minimum at x?

(e) On what interval(s) is f concave down?

(f) What is $\lim_{h\to 0} f(h)/h$?

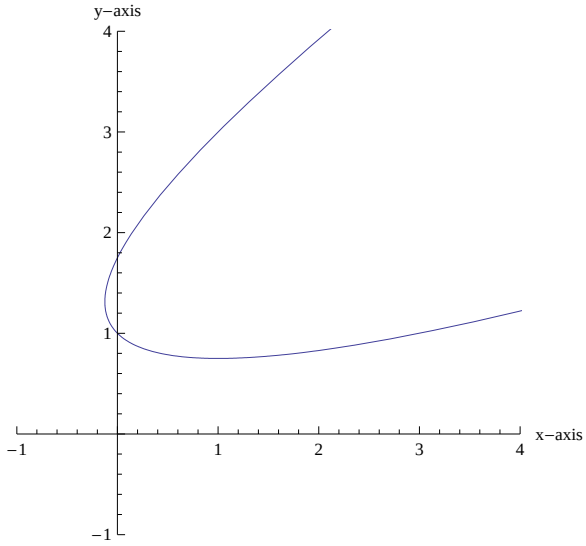
4. (8 points) A UFO flies horizontally at a constant speed at an altitude of 15 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of 0.1 rad/min. How fast is the UFO flying? (Include units in your final answer.)

5. (12 points)

Let C be the curve given by the parametric equations

$$\begin{cases} x(t) &= 2t^2 - t \\ y(t) &= t^2 + t + 1. \end{cases}$$

where $-\infty < t < \infty$.



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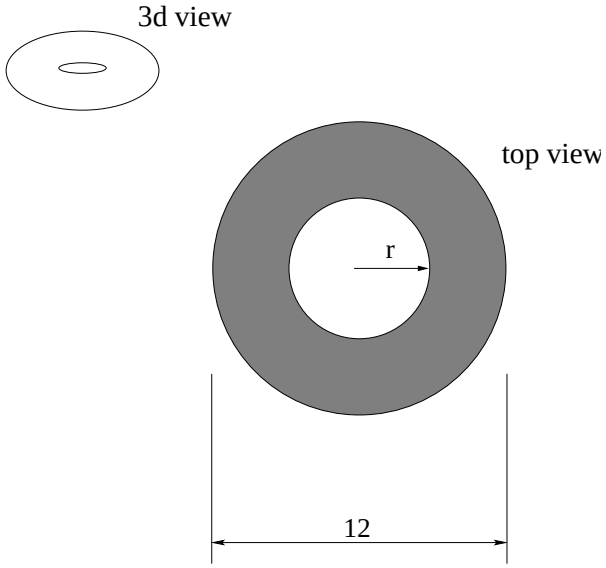
(a) Find all points P on C for which the tangent line to C at P passes through the point $(1, 0)$.

(b) Now suppose that a particle is traveling along the curve C. Its position at time t is given by $(x(t), y(t))$. When is the particle heading directly towards the point $(1, 0)$?

6. (10 points) *AirToys* manufactures flying rings (a type of frisbee in the shape of a donut) with outer diameter 12 inches. The volume of a flying ring with outer diameter 12 inches and inner hole radius r inches can be computed by the formula:

$$V = 2\pi^2 \left(9 - \frac{r^2}{4}\right)\left(3 - \frac{r}{2}\right).$$

The material used by *AirToys* to make these flying rings costs 10 cents per cubic inch. Currently, the rings have an inner radius of 5 inches, so the material needed to manufacture a ring costs about \$2.71 Due to the economic downturn, *AirToys* wants to lower their material costs a little. Use a tangent line approximation to estimate the inner radius r that would result in a cost of \$2.50 per ring.



7. (12 points) Compute the following limits. Your final answer should be a number, $+\infty$, $-\infty$ or "does not exist". Show all steps and justify your answer.

(a) $\lim_{\theta \to 0} \dfrac{\tan(7\theta)}{\sin(2\theta)} =$

(b) $\lim_{y \to \pi^-} \dfrac{\cos y}{\pi - y} =$

(c) $\lim_{x \to \infty} x^{\frac{1}{x}} =$

8. (10 points) Consider the equation $3(x^2 + y^2)^2 = 25(x^2 - y^2)$ that describes the graph of a lemniscate.

   (a) Find $\dfrac{dy}{dx}$ using implicit differentiation.

   (b) Find the equation of the tangent line to the graph of the lemniscate at the point $(-2, -1)$.

9. (14 points) For this problem, consider the function $y = f(x) = \dfrac{x^3}{x^2 - 1}$ on $(-\infty, \infty)$. Please show your work, and leave your answers in exact form (not decimal form).

   (a) Determine any x-intercepts and any vertical asymptotes for the graph of $y = f(x)$.

   (b) Find all of the critical numbers c for $f(x)$.

(problem continues on next page)

9. (continued)

(c) Determine in each case whether there is a local maximum or local minimum at c.

(d) Find the x and y coordinates of all of the inflection points (if any).

(problem continues on next page)

9. (continued)

(e) Sketch the curve. Make sure to label any points on the graph corresponding to local maximum or local minimum values of the function. Also, label any inflection points.