

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around **YOUR FINAL ANSWER** to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	15	
2	10	
3	12	
4	10	
5	10	
6	15	
7	16	
8	12	
Total	100	

1. [15 points total] Find dy/dx for the following.

(a) [5 points] $y = \sqrt{x} - \frac{1}{\sqrt{3x-5}}$

(b) [5 points] $y = \frac{\sin(x^2)}{\cos^3(2x)}$

(c) [5 points] $y = (x^2 + 1)^{(x+1)}$

2. [10 points total] Evaluate the following limits.

(a) [5 points] $\lim_{t \rightarrow \infty} \frac{t^2 + 1}{t \ln t}$

(b) [5 points] $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 1}}{\sqrt{4x^2 - 8}}$

3. [12 points total] A tennis ball is dropped from a height of 10 feet at time $t = 0$ seconds. It bounces up and down for the next 2π seconds according to the following function:

$$s(t) = 10 e^{-2t} \sin^2 \left(t + \frac{\pi}{2} \right) \quad 0 \leq t \leq 2\pi.$$

Where $s(t)$ is the distance of the ball from the ground.

- (a) [4 points] Find **all** the times when the velocity of the ball is zero.
- (b) [6 points] Find all values of t for which $s(t)$ is a local maximum and determine the value of $s(t)$ at those points.

- (c) [2 points] Determine the global maximum of $s(t)$.

4. [10 points] Find the derivative of the function $f(x) = x + \frac{2}{x}$ using the definition of the derivative. **Do not use any differentiation formulas.**

5. [10 points] Use linear approximation to estimate the y value of a point $\left(\frac{99\pi}{100}, y\right)$ on the curve $y = \sin(y + x)$ near $(\pi, 0)$.

6. [15 points total] Consider the function $P(x) = x^2 e^{-x^2+2}$. The domain is all real numbers.

(a) [3 points] Calculate the intervals in which $P(x)$ is increasing and decreasing.

(b) [3 points] Find all local extrema for $P(x)$ and justify your answers.

(c) [3 points] Calculate the intervals where $P(x)$ is concave up and concave down.

(d) [**2 points**] Find all horizontal asymptotes of $P(x)$.

(e) [**4 points**] Sketch the graph of $P(x)$ below, labeling your extrema and indicating any asymptotes and x -intercepts and y -intercepts.

7. [16 points total] To the right is a sketch of the graph of $y = f(x)$ which has domain $(0, 7)$. Note that $x = 7$ is a vertical asymptote.

Take also $g(x) = f(f(x^2 + 3))$.

Estimate the following values; 2 points each, no partial credit.

(a) the domain of $f'(x)$

(b) the domain of $g(x)$

(c) $\lim_{a \rightarrow 3^-} f(a)$

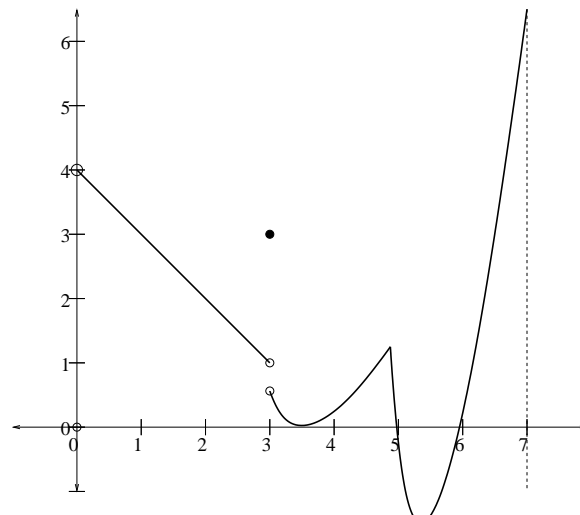
(d) $\lim_{a \rightarrow 3^-} f'(a)$

(e) $f''(1)$

(f) $f(f(f(3)))$

(g) $\lim_{a \rightarrow 7^-} f'(a)$

(h) $g'(1)$



8. [12 points] Starting at point B , an object travels with constant speed 7 cm per second counterclockwise around a circle of radius 10 cm. A light is positioned 20 cm from the center of the circle, directly across from point B . A wall is positioned 30 cm from the center of the circle directly opposite the light. Find the velocity of the shadow cast by the object on the wall as a function of time.

