Math 124AA,AB,BC **First Midterm** **8:30 Tuesday, October 23, 2007**

150 points total

1. (25 points) Define

$$y = f(x) = \begin{cases} 2^{1/x} & \text{if } x < 0; \\ \sqrt{x} & \text{if } x \geq 0. \end{cases}$$

(a) Is this function continuous everywhere? Explain your answer carefully, using complete sentences and solid reasoning. (No credit for an answer without a good explanation.)

(b) Sketch the graph of $y = f(x)$ on the interval $-1 \leq x \leq 1$.

2. (25 points) Interpret the limit

$$\lim_{h \to 0} \frac{\sqrt{25 - (3+h)^2} - 4}{h}$$

as the slope of a certain tangent line on a circle, and evaluate the limit by working directly with the circle. (Almost no algebra is needed for this problem.)

3. (30 points) Let

$$y = f(x) = \frac{2+x}{2-x}.$$

(a) Find the equations of all vertical and horizontal asymptotes.

(b) Find the value $x = a$ for which the graph of $y = f(x)$ crosses the x-axis.

(c) Find the slope of the secant line joining the point where the graph crosses the x-axis (which you found in part (b)) to the point $(a+h, f(a+h))$. Use algebra to simplify your answer.

(d) Write the derivative $f'(a)$ (where a is the x-value found in part (b)) as a limit. What is the value of this limit?

4. (30 points) In the graph of $f(x)$ at the top of the next page, find all x values for which

(a) $f(x)$ is not defined;

(b) $f(x)$ is defined, but is not continuous;

(c) $f(x)$ is defined and continuous, but is not differentiable.

5. (40 points) The 24 graphs on the next page are labeled by letters from (a) to (x). For each of the following graphs of $f(x)$, give the letter of the graph that looks most like it could be the graph of the derivative function $f'(x)$:

(1) x; (2) w; (3) t; (4) s; (5) p; (6) j; (7) g; (8) b

Answers to first midterm for Math 124AA,AB,BC

1. (a) The only place where there could be a discontinuity is at $x = 0$. If we approach $x = 0$ from the left, the formula $2^{1/x}$ means raising 2 to a very large negative power, and this approaches 0. Since $f(0) = \sqrt{0} = 0$, the function is continuous at $x = 0$.

(b)

2. This limit is the slope of the tangent line to the circle $y = \sqrt{25 - x^2}$ at the point $(3, 4)$. Since the radius has slope $4/3$, the tangent (which is perpendicular to the radius) has slope $-3/4$.

3. (a) vertical: $x = 2$, horizontal: $y = -1$; (b) The numerator is zero when $x = a = -2$. (c) The slope from $(-2, 0)$ to $(-2 + h, f(-2 + h))$ is

$$\frac{\frac{2+(-2+h)}{2-(-2+h)} - 0}{h} = \frac{\left(\frac{h}{4-h}\right)}{h} = \frac{1}{4-h}.$$

(d)

$$\lim_{h \to 0} \frac{1}{4-h} = \frac{1}{4}.$$

4. (a) 4; (b) 3, 5, 7; (c) 1, 6.5.

5. (1) k; (2) i; (3) d; (4) f; (5) h; (6) o; (7) t; (8) x