

Midterm 1 - Winter 2003

MATH 124 GH

Quiz section: _____

Instructor: Alicia Cantón

Your TA: _____

Name:

Student ID number:

You will have 80 minutes to complete the test. Please, show all your work. Partial credit will be awarded if you give partial solutions (showing your work). Correct answers with insufficient work will not get all the credit.

Be aware that there are questions in both sides of the pages.

A scientific calculator and one handwritten sheet of notes (8.5×11 in², both sides) are allowed. Graphing calculators are NOT allowed.

Please, turn off your cell phone.

Problem	Possible	Score
1	20	
2	20	
3	20	
4	20	
Total	80	

Name: _____

Problem 1 (20 points)

A crow and a pigeon start flying from the roof of the same house. Both birds fly 50 ft from the ground. The position of the crow in its flight is given by $c(t) = (2t - 1, 4t + 5)$ and the position of the pigeon is given by $p(t) = (t - 1, (t - 2)^2 + 1)$ for $t \geq 0$.

a) (7 points) Find the equations of the trajectories of the crow and the pigeon (that is, the functions $y = f(x)$ and $y = g(x)$ whose graphs give the parametrized curves $c(t)$ and $p(t)$ respectively.)

b) (6 points) Sketch the graph of both trajectories.

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c) (7 points) Both the crow and the pigeon fly through the point $(5, 17)$. Do they collide at that point? In other words, do they arrive at that point at the same time?

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Problem 2 (20 points)

The concentration of carbon 14 decreases exponentially with time. A scientist analyzes a sample of a fossilized tree and observes that its carbon 14 concentration is 2. By comparing with living trees, he knows that 3,000 years ago the concentration of carbon 14 in the same sample was 54.

a) (10 points) Find a formula for the concentration of carbon 14 in the fossil, where the time t is given in thousands years (for example, $t=2$ means 2,000 years) and $t=0$ is the present, the time when the scientist analyzes the sample.

b) (5 points) What would be the concentration of carbon in the sample in 1,000 years?

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c) (5 points) When was the concentration of the sample equal to 10? Please show the work you have done to get your answer.

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Problem 3 (20 points)

Tom throws a ball vertically in the air. The distance from the ball to the ground is given by,

$$d(t) = -t^2 + 10t + 4, \quad 0 \leq t \leq 10,$$

where the distance is in feet and the time is measured in seconds.

a) (6 points) Find the average velocity of the ball during the first 5 seconds after Tom has thrown the ball.

b) (6 points) Calculate the instantaneous velocity of the ball at time t seconds by evaluating the limit $\lim_{h \rightarrow 0} \frac{d(t+h) - d(t)}{(t+h) - t}$. (Do not approximate it using the calculator!).

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c) (6 points) Find a time t for which the (instantaneous) speed of the ball is zero.

d) (2 points) At which time is the ball at its highest position?

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Problem 4 (20 points)

Consider the function $f(x) = \begin{cases} \frac{x^2 + 1}{x - 2}, & x \leq 3 \\ 3x + 1, & x > 3. \end{cases}$

a) (5 points) Find the domain of $f(x)$.

b) (5 points) Find the vertical asymptotes of $f(x)$.

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c) (5 points) Find $\lim_{x \rightarrow 3} f(x)$. Please show your work.

d) (5 points) Is f continuous at $x = 3$? Explain why the function is continuous or why it is not continuous.