KEY Collingwood A014

- no credit without work.
- (-1) if dropped "lim" notation incorrectly

1. (12pts) (4pts each)

Calculate the following limits. In each case, you MUST EXPLAIN your reasoning using techniques developed in the class thusfar. No credit for answers only. Plugging numbers into a calculator is not an acceptable justification.

(a) $\lim_{x\to 1} \frac{4x^2-12x+8}{3x-3} = \lim_{x\to 1} \frac{4(x^2-3x+2)}{3(x-1)}$

$= \underbrace{\lim_{x\to 1} \frac{4(x-2)\cancel{(x-1)}}{3\cancel{(x-1)}}}_{\text{2pts if factored and cancelled}} = \lim_{x\to 1} \frac{4}{3}(x-2) \underbrace{= \frac{4}{3}(1-2)}_{\text{by continuity.}} = -\frac{4}{3}$

1pt if plugged $x=1$ in

1pt if correct ans.

(b) $\lim_{x\to 0} \frac{(\cos x)^2}{x^3-1} = \frac{\cos(0)^2}{0-1} = -1$ — 2pts.

must say why you can plug in $x=0$; the word "continuous" — 2pts

(c) $\lim_{t\to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right) = \lim_{t\to 0} \frac{t - t\sqrt{1+t}}{t^2\sqrt{1+t}}$

$= \lim_{t\to 0} \left(\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\right)\left(\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}\right)$ ← pts if know to rationalize

$= \lim_{t\to 0} \frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})} = \lim_{t\to 0} \frac{-t}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \underbrace{\lim_{t\to 0} \frac{-1}{\sqrt{1+t}\,(1+\sqrt{1+t})}}_{\text{1pt getting to here}} = \underbrace{\frac{-1}{\sqrt{1}\,(1+\sqrt{1})}}_{\text{1pt plug in } t=0} = -\frac{1}{2}$

1pt answer

2. (6pts) Is there a constant c so that

$$\lim_{t\to 0} \frac{\sqrt{c+t}-\sqrt{c}}{t} = 2?$$

If you can find such a constant, find it; if there is no such constant, explain why.

First, calculate limit on left hand side:

$$\lim_{t\to 0} \frac{\sqrt{c+t}-\sqrt{c}}{t} = \lim_{t\to 0} \left(\frac{\sqrt{c+t}-\sqrt{c}}{t}\right)\left(\frac{\sqrt{c+t}+\sqrt{c}}{\sqrt{c+t}+\sqrt{c}}\right)$$

1pt if rationalize

$$= \lim_{t\to 0} \frac{c+t-c}{t(\sqrt{c+t}+\sqrt{c})}$$

1pt to get this

$$= \lim_{t\to 0} \frac{t}{t(\sqrt{c+t}+\sqrt{c})} = \lim_{t\to 0} \frac{1}{\sqrt{c+t}+\sqrt{c}}$$

$$= \frac{1}{\sqrt{c}+\sqrt{c}} = \frac{1}{2\sqrt{c}} \leftarrow \text{1pt}$$

calculating this limit worth 3pts total

Now try to solve:

$$\frac{1}{2\sqrt{c}} = \lim_{t\to 0} \frac{\sqrt{c+t}-\sqrt{c}}{t} = 2 \quad \leftarrow \text{1pt if got this eqn}$$

$$2\sqrt{c} = \frac{1}{2}$$

$$\sqrt{c} = \frac{1}{4}$$

$$c = \frac{1}{16}$$

1pt for solving for c.

So, yes, there is a constant: $c = \frac{1}{16} \leftarrow$ 1pt if got correct ans.