NAME _____ TA'S NAME _____

STUDENT ID _____ SECTION _____

Math 124 (Collingwood)
Autumn 2004

Midterm 1
October 19, 2004

Point totals are indicated. Unless stated otherwise, you must show your work to receive credit.

1. _____ /12

2. _____ /6

3. _____ /10

4. _____ /12

5. _____ /10

Total _____ /50

1. (12pts)

Calculate the following limits. In each case, you MUST EXPLAIN your reasoning using techniques developed in the class thusfar. No credit for answers only. Plugging numbers into a calculator is not an acceptable justification.

(a) $\lim_{x\to 1} \dfrac{4x^2 - 12x + 8}{3x - 3} =$

(b) $\lim_{x\to 0} \dfrac{(\cos x)^2}{x^3 - 1} =$

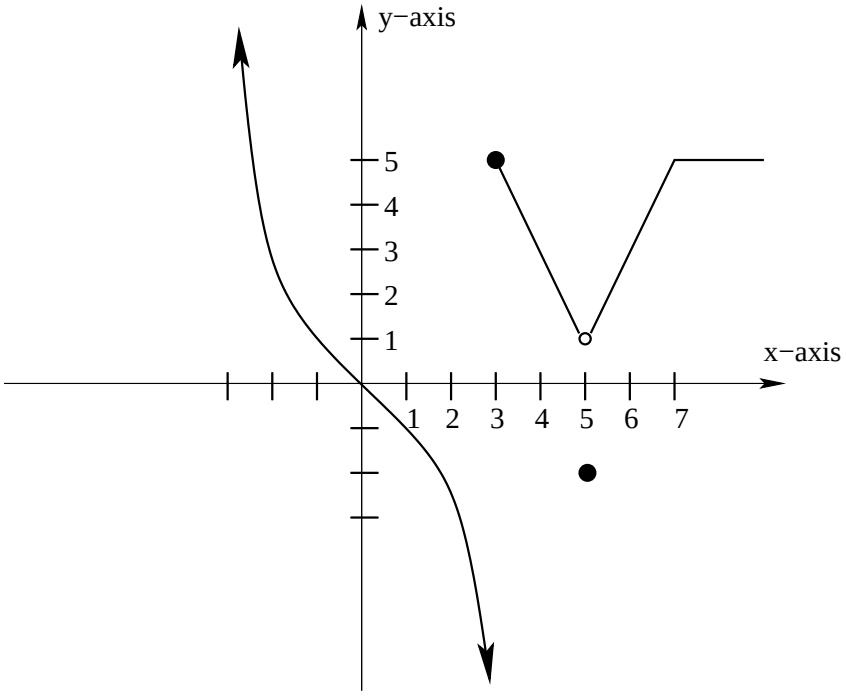
(c) $\lim_{t\to 0} \left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} \right) =$

2. (6pts) Is there a constant c so that

$$\lim_{t\to 0} \frac{\sqrt{c+t} - \sqrt{c}}{t} = 2?$$

If you can find such a constant, find it; if there is no such constant, explain why.

3. (10 pts) Use this graph of the function $y = f(x)$ to answer the following questions. Box your answer. Each part is graded right or wrong, no partial credit. You need not show work on this question.



(a) $\lim_{x \to 5} f(x) =$

(b) $f(5) =$

(c) $\lim_{x \to 0} \dfrac{f(x)}{x} =$

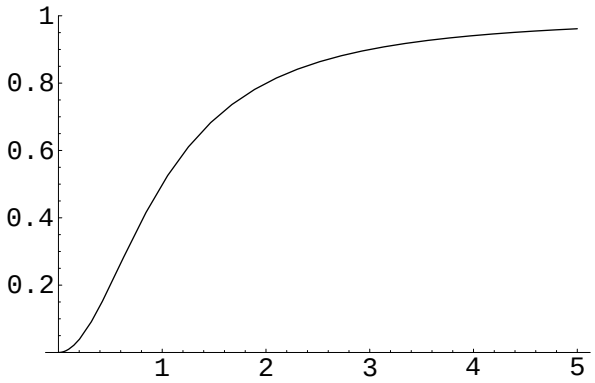
(d) The average rate of change between $x = 5$ and $x = 6$ equals _____

(e) The instantaneous rate of change at $x = 6$ equals _____

4. (12 pts) In this problem we will use the function

$$V(t) = \frac{t^2}{t^2+1}.$$

The graph of this function is pictured.

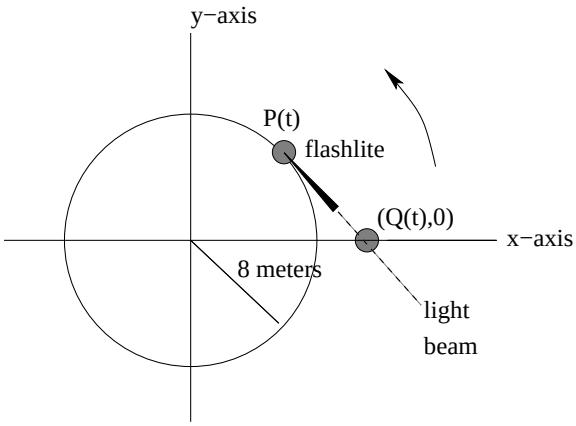


(a) Find the equation of the tangent line to the graph of $V(t)$ at $P = (1, \frac{1}{2})$. You must do this by calculating the limiting value of secant line slopes. You must use limits and our limit rules to arrive at your answer. No credit for shortcut rules you may have learned in high school. Give an exact answer.

(b) If $s(t)$ =the slope of the tangent line at the point $(t, V(t))$, find

$$\lim_{t\to\infty} s(t) =$$

5. (10pts) A flashlight is attached to the edge of a rotating wheel of radius 8 meters. The wheel rotates counterclockwise $\frac{\pi}{3}$ rad/sec and the flashlight begins at the position $(8, 0)$. Assume that the light beam of the flashlight always points in a direction tangential to the circle, as pictured. Let $P(t)$ be the coordinates of the flashlight at time t seconds and $Q(t)$ the place where the light beam crosses the x-axis. Here is a picture of the situation, from above, with a coordinate system imposed.



(a) $\lim_{t \to \frac{3}{2}^-} Q(t) =$

(b) At time $t = 1$ second, the location of the flashlight will be $P(1) = (4, 4\sqrt{3})$. Find where the light beam crosses the x-axis; i.e. find $Q(1)$.