

Math 124
Midterm 1
October 25, 2005

Name: _____

TA/section: _____

Instructions: Unless otherwise stated, you MUST show work for credit. No notes allowed. A scientific calculator is allowed. No graphing calculators. Make sure you have a complete exam consisting of 7 pages.

Any final numerical answer must be either left in “exact form” or you need to give 3 decimal places of accuracy.

SCORING:

1. _____/6

2. _____/5

3. _____/12

4. _____/8

5. _____/14

6. _____/5

/ 50

1. (6 points; 2 points each) Using the **derivative rules** you have learned, find the derivatives. You do not need to simplify your final answer. Show some work. You must **BOX YOUR FINAL ANSWER** to be graded.

(a) $f(x) = 4x^3 - 3x^2 + x - 3$,

$$f'(x) =$$

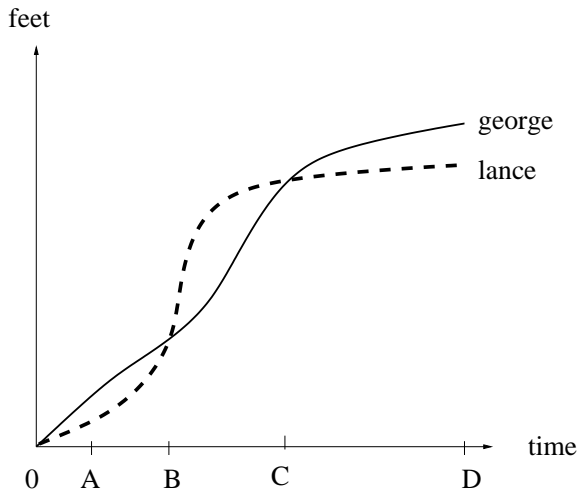
(b) If α is a non-zero constant and $f(x) = \frac{x}{\alpha^2 + 1} + \frac{\alpha}{x^2 + 1}$,

$$f'(x) =$$

(c) If m, b are constants and $y = mx + b$,

$$y'(x) =$$

2. (5 points; 1 point each) Lance and George are racing their bikes. The distance each rider has traveled as a function of time is pictured below. For each question, circle your answer. No partial credit. No need to show your work on this question.



- (a) Who is ahead in the race at time A?

lance george

- (b) Who is going faster at time B?

lance george

- (c) Who is going faster at time C?

lance george

- (d) Suppose the race is over at time D. During the course of the race, who had the largest instantaneous velocity?

lance george

- (e) On the time interval $[A,B]$, who has the largest average velocity?

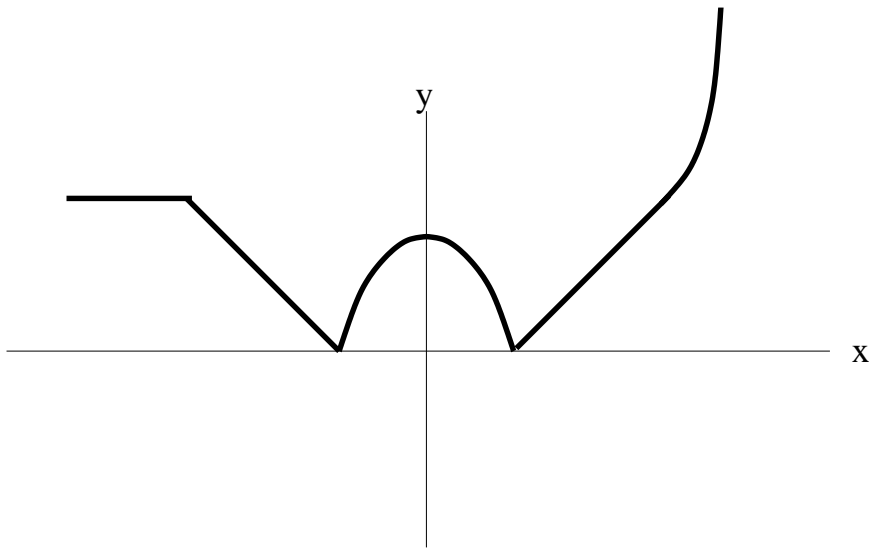
lance george

3. (12 points) This problem will work with the curve $y = f(x) = \sqrt{x+2}$, where the domain is assumed to be all $x > -2$.

(a) Using the **limit definition**, calculate the slope of the tangent line to the curve at the point $P = (a, f(a))$. No credit for the use of “formulas” or “rules” you may have learned in other courses. You must justify your eventual limit calculation and show your work.

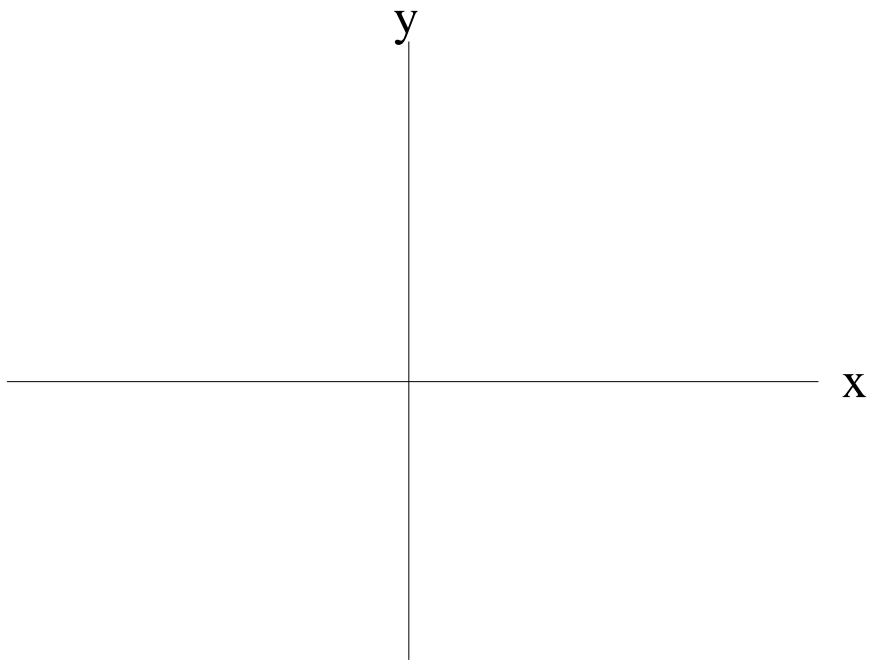
(b) Find the points on the curve where the tangent line passes through the point $(0, 2)$.

4. (8 points) Here is a picture of the graph of a function $y = f(x)$.



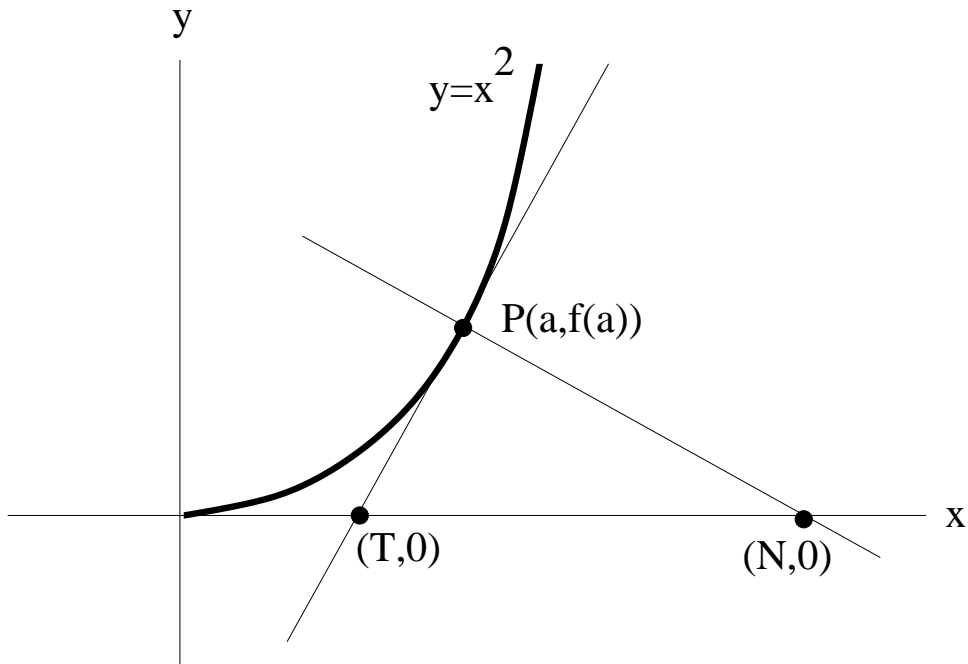
Graph of $y=f(x)$

Sketch a rough graph of its derivative function below. For full credit, you need to highlight important features, such as places the derivative function is zero or undefined.



Graph of $y=f'(x)$

5. (14 points) The picture below shows two lines passing through the point $P = (a, f(a))$ on the curve $y = f(x) = x^2$. One line is the tangent line at P and the other line is perpendicular to the tangent line through P .



- (a) The x -intercepts of the two lines are labeled T and N . Find a formula for T as a function of a . Likewise, find a formula for N as a function of a .

- (b) Find

$$\lim_{a \rightarrow 0^+} \frac{N - T}{N} =$$

$$\lim_{a \rightarrow \infty} \frac{N - T}{N} =$$

6. (5 points) A particle is moving in the xy -plane with parametric equations: $x(t) = 5t^2 + 10t$, $y(t) = te^t$, where t represents time in seconds. When will the tangential line to the path of travel have slope 1?