

Midterm #1 Math 124 Autumn 2005

Name:

Student number:

In the problems below, find slopes using the limit methods from this class. You will only receive partial credit if you use differentiation rules or numerical approximations.

1. Find the equation of the line which is tangent to the graph of $y = 2x^2 + x + 3$ at $(1, 6)$.

2. Suppose f is a differentiable function of t .

a. What is the slope of the secant line between the points $(a, f(a))$ and $(t, f(t))$?

b. Give the definition of the derivative of f at a using only symbols (an equation with no words).

c. Using complete sentences with English words, describe each term in your definition

b. For example, "The left-hand side of the equation above denotes the of f at a . The ratio in the right-hand side of the equation represents the of the line between and The symbol "lim" stands for ... which means"

d. If t is time in minutes and $f(t)$ is distance in feet, what is the average velocity between a and $a + 1$? What are the units of this velocity?

e. What is the instantaneous velocity at $t = a$?

3. A shed with vertical cross-section $0 \leq y \leq 16 - (x/2)^2$, for $-8 \leq x \leq 8$, is held down by a rope going over the top and anchored at $(-10, 0)$ and $(10, 0)$. The rope is on the roof of the shed for $-a \leq x \leq a$.

a. Find a . (note that the rope is tangent to the roof at the point it leaves the roof).

b. Find the multipart function describing the rope.

4. A train follows a straight track, then begins to turn. The front of the train has a path given by $(x(t), y(t))$ where $x(t) = t/4$ and

$$y(t) = \begin{cases} 0 & \text{when } t \leq 0 \\ t^{\frac{3}{2}} & \text{when } t \geq 0. \end{cases}$$

The parameter t is time measured in minutes.

a. Find y as a multipart function of x along the path.

The following might be helpful for part b: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

b. The lights of the train are tangent to the path, and point in a direction given by the angle θ measured counterclockwise from positive horizontal direction. Find $\theta(x)$ as a multipart function of x . (Note that $\tan(\theta)$ is the slope of the curve.)

c. Where is $\theta = \frac{\pi}{4}$? (Give (x, y) coordinates)

d. When does $\theta = \frac{\pi}{4}$ occur? (give the time).