1 (10 pts = 2+3+3+2) Compute the limits. Your final answer should be a number, $+\infty$, $-\infty$, or "does not exist".

a) $\lim_{x\to 2^-} \frac{e^x}{2-x} = "\frac{e^2}{2-2^-}" = "\frac{e^2}{0^+}" = \boxed{+\infty}$

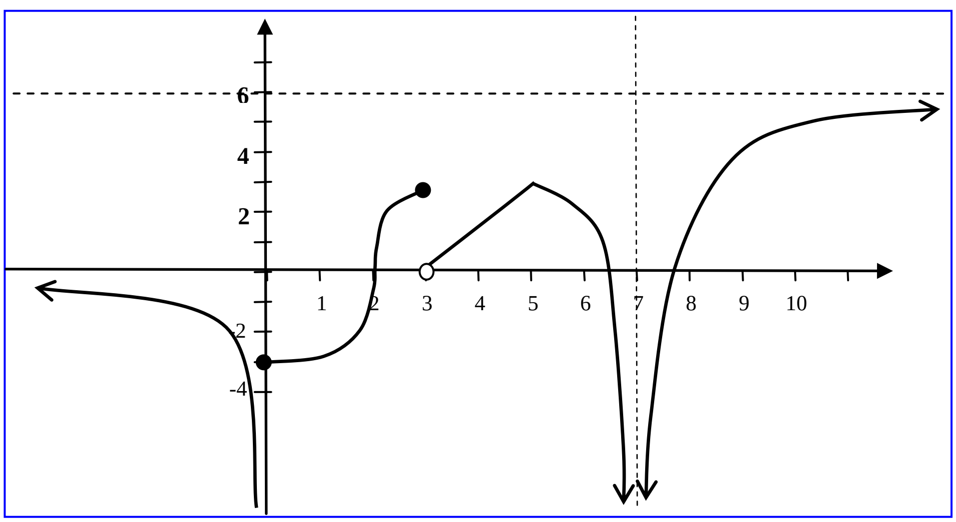
b) $\lim_{x\to\infty}\left(x-\sqrt{x^2+3}\right) = \lim_{x\to\infty}\left(\left(x-\sqrt{x^2+3}\right)\frac{x+\sqrt{x^2+3}}{x+\sqrt{x^2+3}}\right) = \lim_{x\to\infty}\frac{x^2-\left(\sqrt{x^2+3}\right)^2}{x+\sqrt{x^2+3}} =$

$$= \lim_{x\to\infty}\frac{x^2-(x^2+3)}{x+\sqrt{x^2+3}} = \lim_{x\to\infty}\frac{-3}{x+\sqrt{x^2+3}} = "\text{-}3/\infty" = \boxed{0}$$

c) $\lim_{t\to 3}\frac{\frac{1}{3}-\frac{3}{t^2}}{t-3} = \lim_{t\to 3}\frac{\frac{t^2-9}{3t^2}}{t-3} = \lim_{t\to 3}\frac{t^2-9}{3t^2(t-3)} = \lim_{t\to 3}\frac{t+3}{3t^2} = \frac{3+3}{3\times 9} = \boxed{\frac{2}{9}}$

d) $\lim_{\theta\to 0}\frac{\sin(13\theta)}{5\theta} = \lim_{\theta\to 0}\left(\frac{\sin(13\theta)}{5\theta}\times\frac{13}{13}\right) = \lim_{\theta\to 0}\frac{\sin(13\theta)}{13\theta}\times\lim_{\theta\to 0}\frac{13}{5} = 1\times\frac{13}{5} = \boxed{\frac{13}{5}}$

2 (12 pts=3+3+2+4) Below is the graph of a function, $y = f(x)$. Use it to answer the following questions, no justification needed:



a) State all the values a for which $f(x)$ is not continuous at $x=a$.

$$a = 0\ (infinite),$$
$$a = 3 (jump)$$
$$a = 7\ (infinite)$$

b) State all the values a for which $f(x)$ is not differentiable at $x=a$.

$$a = 0, 3, 7\ (discontinuous)$$

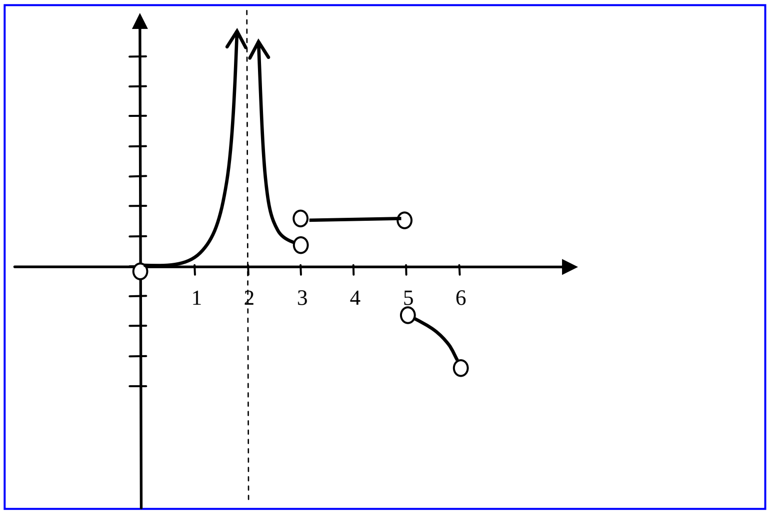
$$a = 2\ (vertical\ tangent)$$

$$a = 5\ (corner)$$

c) Evaluate the two limits at infinity:

$\lim_{x\to+\infty} f(x) = 6$ and $\lim_{x\to-\infty} f(x) = 0$

d) Sketch the portion of the graph of the derivative function $f'(x)$ corresponding to the interval $0 \le x \le 6$.



3 (8 points) Let $g(x)=\dfrac{x^2-2x-8}{x+2}$. Compute all the appropriate limits to determine each of the following:

a) Determine if this function has any horizontal asymptotes. If it does, list the equation(s). If none, say so.

$$\lim_{x\to\infty}\frac{x^2-2x-8}{x+2}=\lim_{x\to\infty}\frac{x-2-\frac{8}{x}}{1+\frac{2}{x}}=\text{"}\frac{+\infty-2-0}{1+0}\text{"}=+\infty$$

$$\lim_{x\to-\infty}\frac{x^2-2x-8}{x+2}=\lim_{x\to-\infty}\frac{x-2-\frac{8}{x}}{1+\frac{2}{x}}=-\infty$$

Since both limits at infinity are infinite, there are no horizontal asymptotes.

b) Determine if this function has any vertical asymptotes. If it does, list the equation(s). If none, say so.

The only possible vertical asymptote can be x=-2. Computing the limit:

$$\lim_{x\to-2}\frac{x^2-2x-8}{x+2}=\lim_{x\to-2}\frac{(x-4)(x+2)}{(x+2)}=-2-4=-6.$$

Since the limit is finite, there is no vertical asymptote (the function has a removable discontinuity)

4 (5 pts) Let $(x)=\begin{cases}\cos x+1, & x\le 0\\ 2-3x, & x>0\end{cases}$. Determine if this function is continuous at $x=0$. Show all work.

We need to check whether $\lim_{x\to0^-} f(x)=\lim_{x\to0^+} f(x)=f(0)$.

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}(\cos x+1)=\cos 0+1=2$$

$$\lim_{x\to0^+} f(x)=\lim_{x\to0^+}(2-3x)=2$$

Since $\lim_{x\to0^-} f(x)=\lim_{x\to0^+} f(x)=f(0)=2$, the function is continuous at x=0.

5 (8 points) Let $f(t) = \mathrm{e}^t \cos t + 1$.

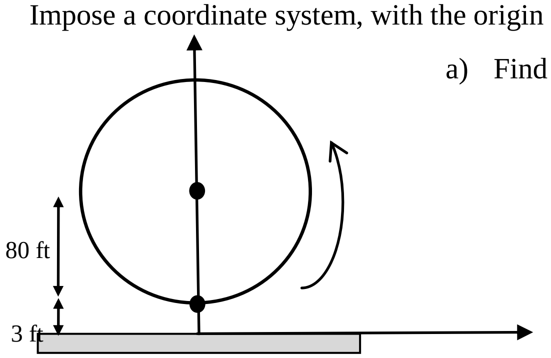
a) Compute its derivative: $f'(t) = (e^t \cos t)' + (1)' = (e^t)' \cos t + e^t (\cos t)' + 0 = \boxed{e^t \cos t - e^t \sin t}$

b) Compute the equation of the tangent line to the graph of $y = f(t)$ at the point (0, 2).

$$m = f'(0) = e^0 \cos 0 - e^0 \sin 0 = 1$$

Using point-slope: $y - 2 = 1(x - 0)$, so the equation of the tangent line at (0, 2) is $\boxed{y = x + 2}$

6 (7 points) Ryan is riding a ferris wheel of radius 80 feet, as pictured below. He starts at the lowest position on the wheel, which is 3 feet above ground. The wheel rotates counterclockwise, at an angular velocity of $\frac{2\pi}{5}$ radians/minute. Impose a coordinate system, with the origin at the point directly below the wheel, at ground level.



a) Find the parametric equations for Ryan's position, $(x(t), y(t))$, after t minutes.

$$\begin{cases} x(t) = 80 \cos\left(\frac{2\pi}{5} t - \frac{\pi}{2}\right) \\ y(t) = 80 \sin\left(\frac{2\pi}{5} t - \frac{\pi}{2}\right) + 83 \end{cases}$$

(The initial angle θ_0 can be taken to be $\frac{-\pi}{2}$, $\frac{3\pi}{2}$, etc)

b) Find the first time when Ryan's horizontal velocity is zero.

Method 1: Considering his starting point and direction of movement, Ryan will have zero horizontal velocity the first time when he gets to the leftmost point, i.e. after he travels $\frac{\pi}{2}$ radians. Solve for t:

$$\frac{\pi}{2} radians = \left(\frac{2\pi}{5}\ rad/min\right) \times (t\ min)$$

Get $\boxed{t = \frac{5}{4}\ minutes}$.

Method 2: Set the horizontal velocity $x'(t) = 0$ and find the earliest solution. Need to use the chain rule.

$$x'(t) = \left(80 \cos\left(\frac{2\pi}{5} t - \frac{\pi}{2}\right)\right)' = -80 \sin\left(\frac{2\pi}{5} t - \frac{\pi}{2}\right) \times \frac{2\pi}{5} = 0$$

$$\sin\left(\frac{2\pi}{5} t - \frac{\pi}{2}\right) = 0 \Rightarrow \frac{2\pi}{5} t - \frac{\pi}{2} = 0 \Rightarrow \boxed{t = 5/4}$$