NAME: ________________________________ Student ID #: _________________

QUIZ SECTION:______

Math 124
Midterm I
April 18, 2006

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Total:	**50**	

- Your exam should contain 5 pages total and 4 problems. Please check your test for completeness.
- You **must explain how you get your answers using techniques developed in this class so far**. Answers with no supporting work, obtained by guess-and-check, or via methods you learned in high school or in other classes will result in little or no credit, even if correct.
- Indicate your **final answer** by placing a box around it.
- Unless otherwise indicated, **leave your answers in exact form** instead of a decimal approximation. That is, $\sqrt{2}$ instead of 1.4142, and $\frac{\pi}{2}$ instead of 1.57. Simplify all you can.
- If you need more room use the backs of pages, but indicate to the reader that you have done so.
- Raise your hand if you have any questions.

GOOD LUCK!

Do you want me to post your grades so far on the class website under the last 4 digits of your student ID?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grades so far.*

1.(12 points) Evaluate the following limits, if they exist:

a) $$\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}=\lim_{h\to 0}\frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)}=\lim_{h\to 0}\frac{(1+h)-1}{h(\sqrt{1+h}+1)}=\lim_{h\to 0}\frac{1}{(\sqrt{1+h}+1)}=\frac{1}{2}$$

b) $$\lim_{x\to\infty}\frac{-x^2+2x+2}{3x+5}=\lim_{x\to\infty}\frac{(-x^2+2x+2)\frac{1}{x}}{(3x+5)\frac{1}{x}}=\lim_{x\to\infty}\frac{-x+2+\frac{2}{x}}{3+\frac{5}{x}}=\frac{-\infty+2+0}{3+0}=-\infty$$

c) $$\lim_{x\to 2}\left(e^x\arccos\left(\frac{x}{4}\right)\right)=e^2\arccos\left(\frac{1}{2}\right)=e^2\left(\frac{\pi}{3}\right)$$

2. (12 points) Let $f(x)=\begin{cases} -3\sin x, & x \le 0 \\ \ln x, & 0 < x \le 1 \\ (x-1)^2, & x > 1 \end{cases}$

a) Does $\lim_{x\to 0} f(x)$ exist? If it does, evaluate it. If it doesn't, justify why not.

Compute the one-sided limits and see if they exist and match each other:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-3\sin x) = -3\sin 0 = 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(\ln x) = -\infty$$

Since $\lim_{x\to 0^+} f(x) = -\infty$, $\boxed{\lim_{x\to 0} f(x) \text{ does not exist}}$.

b) Does $\lim_{x\to 1} f(x)$ exist? If it does, evaluate it. If it doesn't, justify why not.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(\ln x) = \ln 1 = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x-1)^2 = (1-1)^2 = 0$$

Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 0$, $\boxed{\text{the limit exists and it's 0}}$.

c) At what values of x is $f(x)$ discontinuous? Justify your answer and state what type of discontinuity does it have at each point of discontinuity: removable, jump, or infinite?

Since *f(x)* is defined and equal to one of the functions we know are continuous whenever defined, the only possible discontinuities are at the points between the different cases, that is at x=0 and x=1.

We saw in part (a) that $\lim_{x\to 0} f(x)$ does not exist, since $\lim_{x\to 0^+} f(x) = -\infty$. This shows $\boxed{\text{that } f(x) \text{ has an infinite discontinuity at x=0}}$.

We saw in part (b) that $\lim_{x\to 1} f(x) = 0$. Since $f(1) = 0 = \lim_{x\to 1} f(x)$, the function is continuous at x=1.

3. (12 points) Let $f(x)=\dfrac{x^2+x-6}{x^2-5x+6}$

a) What is the domain of *f(x)*?

Since *f(x)* is a rational function, it is defined for all x except at the roots of the denominator.

$x^2-5x+6=(x-3)(x-2)=0$ when x=3 or x=2.

The domain is of *f(x)* is all x, except x=2 & x=3.

c) Find all the vertical asymptotes of *f(x)*. Show all work.

Vertical asymptotes are the vertical lines x=a, where $\lim_{x\to a} f(x)=\pm\infty$. Since *f(x)* is a rational function, the only such possible points are the values of x for which the denominator is zero (x=2, x=3). We need to check these two limits to see if the function has or not vertical asymptotes atthese two values of x.

First, since $f(x)=\dfrac{x^2+x-6}{x^2-5x+6}=\dfrac{(x+3)(x-2)}{(x-3)(x-2)}$, we see that at all x except at x=2, $f(x)=\dfrac{x+3}{x-3}$.

We can then compute $\lim_{x\to 2} f(x)=\lim_{x\to 2}\dfrac{x+3}{x-3}=\dfrac{2+3}{2-3}=\dfrac{5}{-1}=-5$. Since the limit is finite, x=2 is not a vertical asymptote.

On the other hand, $\lim_{x\to 3} f(x)$ does not exist, and $\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\dfrac{x+3}{x-3}\overset{\frac{6}{0^-}}{=}-\infty$, $\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}\dfrac{x+3}{x-3}\overset{\frac{6}{0^+}}{=}+\infty$

Since at least one of the one-sided limit is infinite, the line x=3 is a vertical asymptote.

d) Find all the horizontal asymptotes of *f(x)*. Show all work.

Horizontal asymptotes are lines y=L such that $\lim_{x\to\pm\infty} f(x)=L$. Hence we have to check the limits at infinity and see if they're finite.

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{(x^2+x-6)\frac{1}{x^2}}{(x^2-5x+6)\frac{1}{x^2}}=\lim_{x\to\infty}\frac{1+\frac{1}{x}-\frac{6}{x^2}}{1-\frac{5}{x}+\frac{6}{x^2}}=\frac{1+0+0}{1+0+0}=1.$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{(x^2+x-6)\frac{1}{x^2}}{(x^2-5x+6)\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{1+\frac{1}{x}-\frac{6}{x^2}}{1-\frac{5}{x}+\frac{6}{x^2}}=\frac{1+0+0}{1+0+0}=1.$$

So the line y=1 is a horizontal asymptote at both positive and negative infinity.

4. (14 points=4+4+4+3+3) According to US Census Bureau (http://www.census.gov/statab/www/statepop.html), the resident population of Washington State was about 5.89 million people in 2000, and about 6.29 million people in 2005. Let $W(t)$ denote the resident population of Washington State t years after the year 2000. Assume that the population can be modeled by an exponential function of the form $W(t)=Ce^{at}$.

a) Find this function $W(t)$. Compute the constants C and a to at least 5 decimal digits.

We're given two points on this function: $W(0)$=5.89& $W(5)$=6.29. We use them to compute our constants:

$W(0)=C=5.89$

$W(5)=5.89e^{a5}=6.29$, so $a=\ln(6.29/5.89)/5 \approx 0.01314$

Hence $\boxed{W(t) \approx 5.89\, e^{0.01314\, t}}$

b) According to this model, in what year will the state's resident population be double that of the year 2000?

Set up the equation: W(t)=2W(0) and solve for t:

$2(5.89)= 5.89\, e^{0.01314(t)}$

$2= e^{0.01314(t)}$

$\ln(2) =0.01314\, t$

$t \approx 52.75$, i.e. $\boxed{\text{in the year 2052}}$

(We also accepted 2053 as correct, since the question is ambiguous as to whether the census is done at the beginning or at the end of the year.)

c) Find a formula in terms of h for the average rate of population growth (average rate of change of W(t)) from the year 2000 until h years later.

$$\frac{W(0+h)-W(0)}{h} = \frac{5.89e^{0..01314\,(0+h)} - 5.89e^{0..01314(0)}}{h} = \frac{5.89(e^{0..01314\,h} - 1)}{h}$$

d) Draw the graph of 3W(1-t). Label the coordinates of at least one point on the graph.

$$3\mathrm{W}(1-\mathrm{t}) = 17.67\mathrm{e}^{0.01314(1-\mathrm{t})}$$