Math 124
Midterm 2
November 22, 2005

Name: _____

TA/section: _____

Instructions: Unless otherwise stated, you MUST show work for credit. No notes allowed. A scientific calculator is allowed. No graphing calculators. Unless instructed otherwise, you can leave your answers in either exact form or use three decimal places of accuracy.

SCORING:

1. ____/12

2. ____/9

3. ____/10

4. ____ /8

5. ____/11

/ 50

1. (12 points; 3pts each) Using the **derivative rules** you have learned, compute the derivatives. You do not need to simplify your final answer. You must BOX YOUR FINAL ANSWER.

   (a) $y = \sin^{-1}(\ln(x^2 + 1))$,

   $y' =$

   (b) $y = \sin^2(3t^2 - t + 1)$,

   $\dfrac{dy}{dt} =$

1. (cont.)

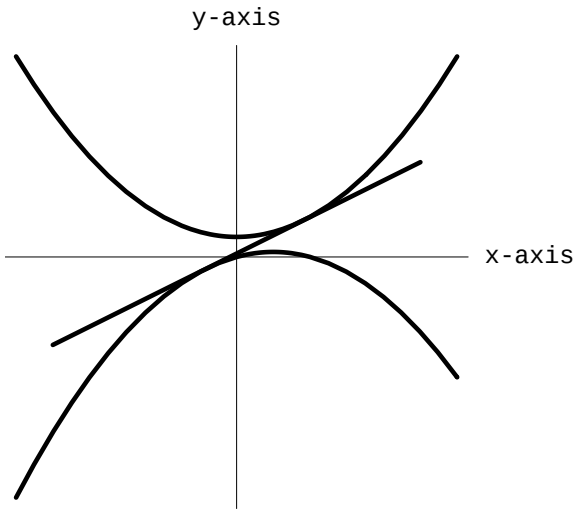
(c) If $f(x) = \sqrt{x + \sqrt{2x}}$,

$f'(x) =$

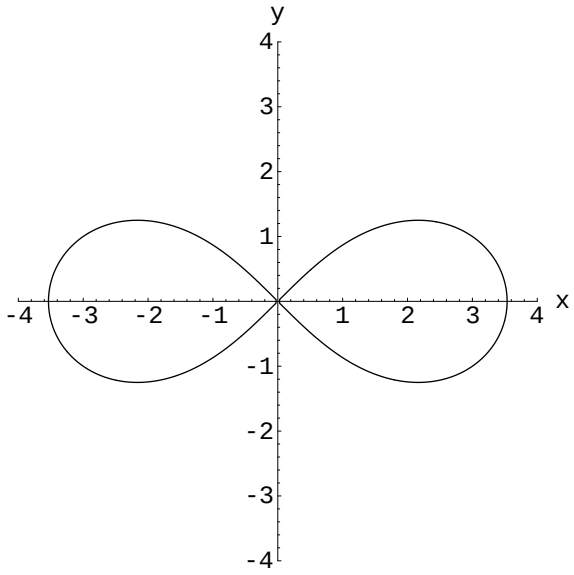
(d) $y = x^{\ln(x)}$,

$y'(x) =$

2. (9 points) The graphs of $f(x) = x^2 + 1$ and $g(x) = -x^2 + x$ are pictured below. Find the equation of the pictured line which is simultaneously tangent to both curves.



3. (10 points) The graph of the equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is pictured below.



(a) (6pts) Find the equation of the tangent line to the curve at $(3, 1)$.

(b) (4pts) Let $Q = (a, 1.01)$ be the point on the curve in the first quadrant with y-coordinate 1.01. Using linear approximation, estimate the value of a. Leave your answer in exact form.

4. (8 points) Sand is being dumped from a conveyor belt at a rate of 2 m^3/min and forms a right circular cone. Assume the radius of the cone is always three times as large as the height of the cone. (Recall, the volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base and h is the height of the cone.) Find the rate at which the height of the cone is increasing when the height is $9m$.

5. (11 points) A particle is moving in the xy-plane with parametric equations

$$\begin{aligned} x(t) &= e^t + e^{-t} \\ y(t) &= e^{-t} \end{aligned}$$

at time t seconds, $t \geq 0$. The units on each axis are centimeters (cm). Recall that the speed of the particle is given by the formula

$$s(t) = \sqrt{(x'(t))^2 + (y'(t))^2}.$$

This problem studies the values of the speed on the time interval [0,1].

(a) (3pts) Find $s(t)$ as an explicit function of t.

(b) (2pts) Calculate $s(0)$ and $s(1)$.

(c) (6pts) Find the critical numbers for $s(t)$ and the minimum speed on the time interval [0,1].