

NAME \_\_\_\_\_ TA'S NAME \_\_\_\_\_

STUDENT ID \_\_\_\_\_ SECTION \_\_\_\_\_

MATH 124H  
AUTUMN 2002

MIDTERM 2  
NOVEMBER 19, 2002

Point totals are indicated in parentheses. You must show your work to receive credit.

- (9) 1. Compute the derivative of the following functions. You need not simplify your answers.

(a)  $y = \left(\frac{x}{x^2 + 1}\right)^4$

(b)  $y = \sin^2(\cos(x^2))$

(c)  $y = x^{(e^x)}$

(8) 2. The function  $y = y(x)$  is defined implicitly by

$$xy^3 - y = 1$$

and satisfies  $y = 1$  when  $x = 2$ . Compute  $y''(2)$ .

- (12) 3. Water is being pumped into an inverted conical tank at a variable rate. The tank has height 8 meters and the diameter at the top is 6 meters. If water is being pumped into the tank at the rate of  $2 \text{ m}^3/\text{min}$ , at the instant when the height of the water is 3 meters, find the rate at which the height is increasing at that instant. Give an exact answer.

- (11) 4. A rat is running counterclockwise around the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

Its position at time  $t$  is given by

$$(x(t), y(t)) = (2 \cos(4\pi t), 3 \sin(4\pi t)).$$

where units of length are in feet and units of time are in minutes. The rat's tail is 1 ft. long and is assumed to be a straight line tangent to the ellipse. (See figure.)

- (a) Write down a formula for the position of the tip of the rat's tail  $(u(t), v(t))$  at time  $t$ , valid for  $0 \leq t \leq \frac{1}{8\pi}$ .
- (b) Compute  $u'(t)$ .

