

MIDTERM 2

Math 124 (Collingwood)
February 25, 2003
100 points

Name _____
Section/TA _____

INSTRUCTIONS: PLEASE READ THIS FIRST:

1. Make sure you have a complete exam. The exam has 9 pages, counting this cover page.
2. Show all your work. NO CREDIT for answers only.
3. No notes allowed.
4. Scientific calculators are allowed. No graphing calculators are allowed.
5. You have 80 minutes.
6. Unless told otherwise, leave answers in EXACT form. For example, $\sqrt{2}$ is exact, whereas 1.414 is an approximation of $\sqrt{2}$.
7. If you have questions, raise your hand and your TA will assist you as soon as possible.

Good Luck.

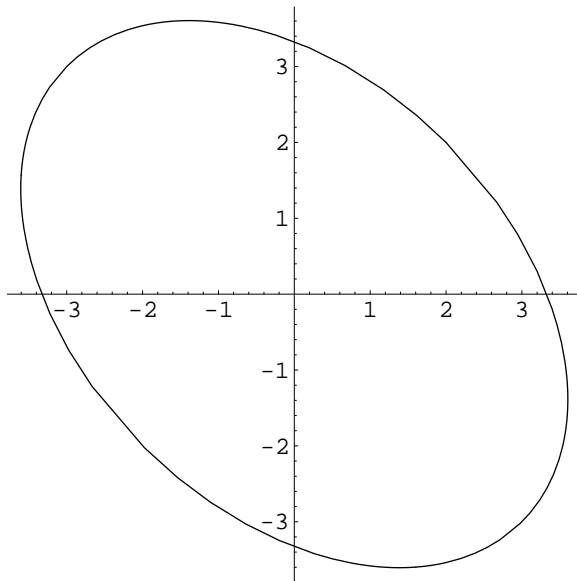
Problem	Total Points	Score
1	18	
2	24	
3	8	
4	8	
5	20	
6	22	
Total	100	

1. (18pts) A moving object has parametric equations:

$$x(t) = 2 \cos(\pi t) - 3 \sin(\pi t),$$

$$y(t) = 2 \cos(\pi t) + 3 \sin(\pi t),$$

where t is in units of seconds. Let $P(t) = (x(t), y(t))$ be the location of the object at time t . The path traced out by the object is pictured below. Answer these questions. You must show work for any credit; no credit for answers only. Make sure to BOX your final answers.



(a) (3pt) Find the coordinates of the location of the object at time $t = 1/2$ second.

$$P(1/2) =$$

Mark this location " $P(1/2)$ " on the picture.

(b) (6pts) Compute the horizontal and vertical velocities of the object at time t .

1. cont.

(c) (4pt) What is the slope of the curve at the location $P(1/2)$?

(d) (5pts) Find the first time the object is located at a position having a horizontal tangent line.

2. (24pts; 8pts each) Compute the derivative. You MUST show work for credit. Box your final answer. You do not need to simplify. Your answers should not involve any uncomputed derivatives..

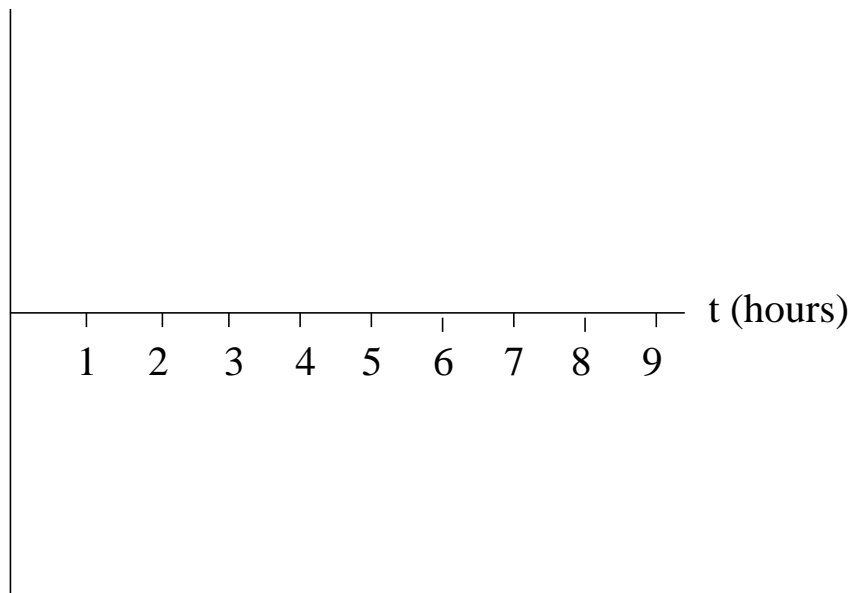
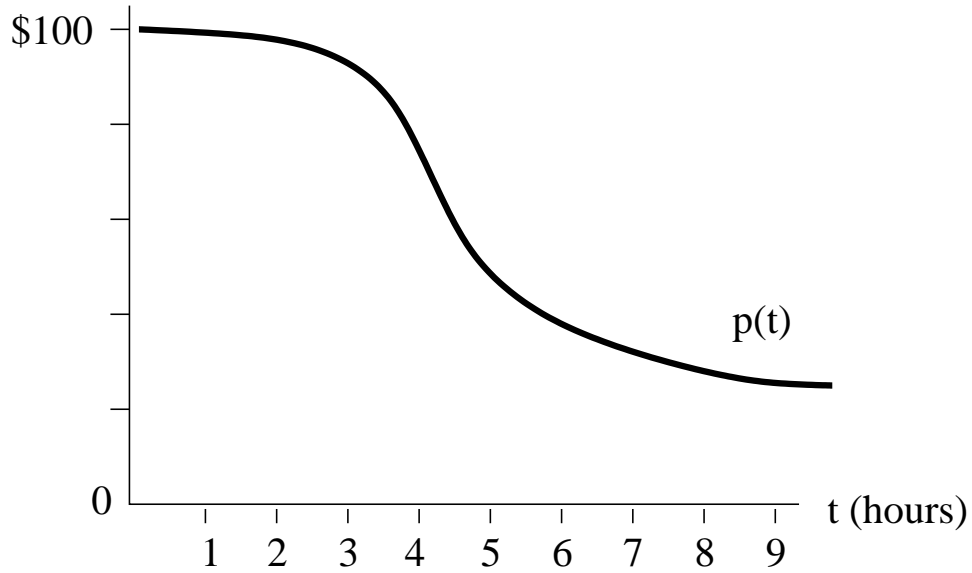
(a) If $y = \cos(e^{-x^2})$, then $\frac{dy}{dx} =$

(b) If a is a constant and $y(t) = t^{\sin(2at)}$, then $y'(t) =$

(c) If $f(x) = \frac{x}{\sqrt{1-x^2}}$, then $f'(x) =$

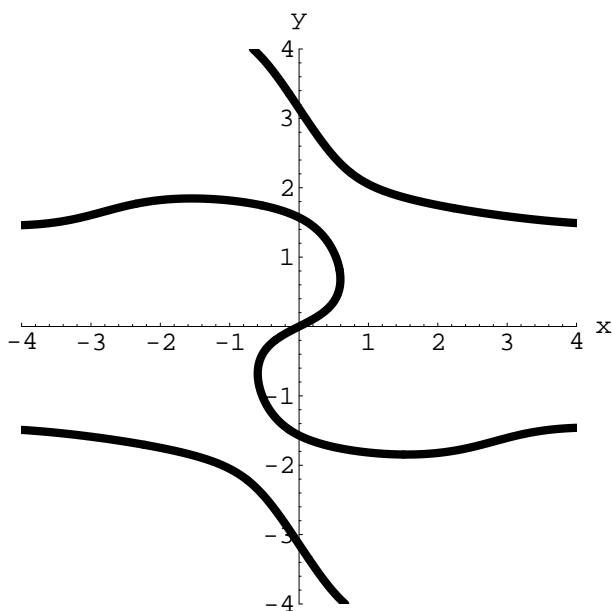
3. (8pts) An object is moving along a straight line. The position of the object on this line at time t seconds is given by the function $p(t) = t^3 - 3t^2 + 6t + 10$ feet. When does the object have the smallest velocity? Where is the object located when this occurs?

4. (8pts) The price of a new internet stock during a 9 hour trading period is given by the function $p(t)$ graphed below. Sketch the graph of $p'(t)$ on the axis provided below. On your derivative plot, indicate the time at which the stock is losing value the fastest.



5. (20pts) Below is a picture of a portion of the graph of the equation:

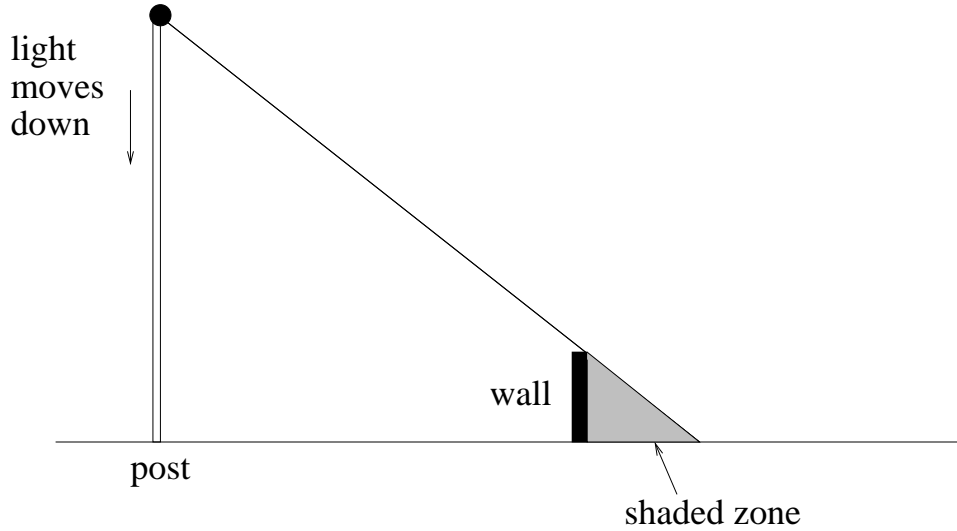
$$\sin(x + 2y) = 2x \cos(y).$$



(a) (10pts) Find $\frac{dy}{dx}$

(b) (10pts) Write the equation of the tangent line to this curve at the origin $(0, 0)$ and sketch this tangent line in the picture.

6. (22pts) A light is located on top of a post that is 40 ft. tall. The post is 40 feet away from a wall that is 6 feet high. Assume the light begins sliding down the post at a rate of 4 ft/sec and remains illuminated as it descends. This causes the shaded zone beyond the wall to increase in size. See picture.



- (a) (4pts) Before the light begins its descent, what is the length of the shaded zone in the picture?
- (b) (4pts) Find an equation relating the height h of the light and the length s of the shaded zone.
- (c) (4pts) Find an equation relating the rate of change of h and the rate of change of s .

6.cont.

(d) (6pts) At time 2 seconds, what is the rate of change of the length of the shaded zone? (Include units.)

(e) (4pts) Does the rate of change of s ever exceed 1600 ft/sec? If so, determine when this happens. If not, explain why not.