NAME: ________________________________ Student ID #: ________________

QUIZ SECTION:______

Math 124
Midterm II

May 16, 2006

Problem 1	13	
Problem 2	8	
Problem 3	6	
Problem 4	7	
Problem 5	16	
Total:	**50**	

- Your exam should contain 5 pages total and 5 problems. Please check your test for completeness.
- Unless otherwise instructed, you **must show how you get your answers, using techniques developed in this class so far**. Answers with no supporting work, obtained by guess-and-check, or via methods you learned in high school or in other classes will result in partial credit at best, even if correct.
- Indicate your **final answer** by placing a box around it.
- **Leave your answers in exact form** instead of a decimal approximation. That is, $\sqrt{2}$ instead of 1.4142, and $\frac{\pi}{2}$ instead of 1.57. Simplify all you can.
- If you need more room use the backs of pages, but indicate to the reader that you have done so.
- Raise your hand if you have any questions.

GOOD LUCK!

Do you want me to post your grades so far on the class website under the last 4 digits of your student ID?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grades so far.*

1. (13 points) Differentiate the following functions. **Do not simplify**.

a) $f(x) = (x^2 + 5)(\sqrt{x} + 8)$ Product Rule, etc

$$\boxed{f'(x) = (2x)(\sqrt{x}+8) + (x^2+5)\left(\frac{1}{2\sqrt{x}}\right)}$$

b) $y = \dfrac{e^x}{\cos x + 3}$ Quotient Rule, etc

$$\boxed{\frac{dy}{dx} = \frac{e^x(\cos x+3) - e^x(-\sin x)}{(\cos x+3)^2}}$$

c) $h(t) = \tan^3(t) + \ln(5t+1) + 10 \arcsin t$ Chain Rule, etc

$$\boxed{h'(t) = 3\tan^2 x \sec^2 x + \frac{5}{5t+1} + \frac{10}{\sqrt{1-x^2}}}$$

2. (8 pts) Find the equation of the tangent line to the elliptic curve $y^2 + y = x^3 - x^2$ at the point (1,0).

Use implicit differentiation to compute $\frac{dy}{dx}$

$$2yy' + y' = 3x^2 - 2x$$

$$y'(2y+1) = 3x^2 - 2x$$

$$y' = \frac{3x^2-2x}{2y+1}$$

Evaluate at (1,0) to get the slope at that point:

$$m = \frac{3\cdot1-2\cdot1}{2\cdot0+1} = 1$$

Use slope-point equation of the line: $y-0=1(x-1)$

Tangent line: $\boxed{y = x-1}$

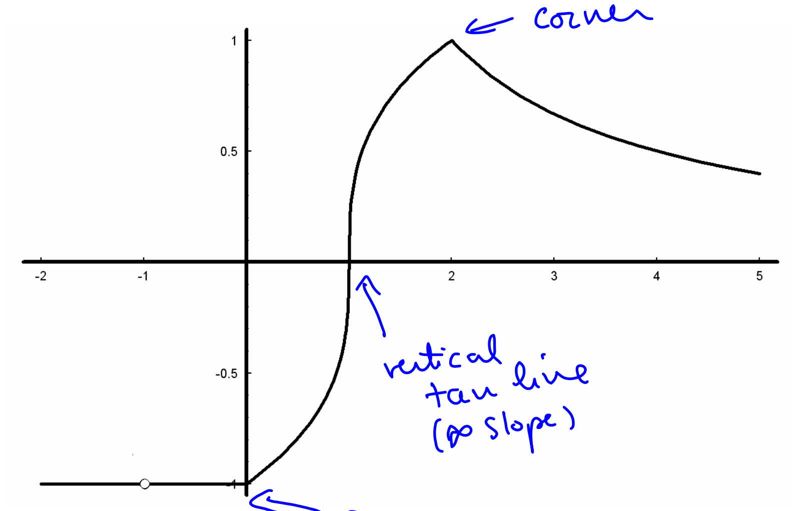
3. (6 points)

The graph on the right is the graph of

$$g(x)=\begin{cases} -1, & x\le 0,\quad x\ne -1 \\ \sqrt[3]{x-1}, & 0<x<2 \\ \dfrac{2}{x}, & x\ge 2 \end{cases}$$

a) List **all** the points at which g(x) is **not** differentiable. No need to justify your answers.

$\boxed{x=-1,\ 0,\ 1,\ 2}$



corner

vertical tan line (∞ slope)

corner

discont. so not diff.

b) What is the equation of the tangent line to the graph of g(x) at x=1? No need to justify your answer.

vertical line thru $x=1$ has equation $\boxed{x=1}$

4. (7 points) An oil tanker is leaking onto the ocean surface, forming a circular oil slick about 0.005 meters thick. The volume of oil spilled is given by the formula $V=\pi h r^2$, where r is the radius of the slick and h is the thickness. Suppose the tanker is leaking at a constant rate of 20 m^3/hr. Find the rate at which the radius of the oil slick is growing when the radius is 50 m (include <u>units</u>).

$h=0.005 \implies V=0.005\pi r^2$

$$\frac{dV}{dt}=0.005\pi(2r)\frac{dr}{dt}$$

at $r=50$: $\quad 20=0.005\pi(2\cdot 50)\left.\frac{dr}{dt}\right|_{r=50}$

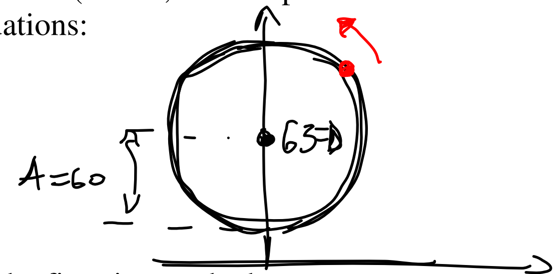
$$\boxed{\left.\frac{dr}{dt}\right|_{r=50}=\frac{40}{\pi}\ m/hr}$$

5. (16 points)

You are on a ferris wheel. As the wheel rotates, your coordinates (in feet, with respect to the base of the wheel) after t seconds are given by the parametrized equations:

$$x(t) = 60\cos\left(\frac{\pi}{30}(t-15)\right)$$

$$y(t) = 60\sin\left(\frac{\pi}{30}(t-15)\right) + 63$$



a) When will you reach the topmost point on the wheel for the first time and what are your coordinates at that point?

One method: topmost point is where y is max

i.e $\sin\left(\frac{\pi}{30}(t-15)\right) = 1$

i.e $\frac{\pi}{30}(t-15) = \frac{\pi}{2} \Rightarrow t-15=15 \Rightarrow \boxed{t=30\text{ sec}}$

Coordinates: $\boxed{(x(30), y(30)) = (0, 123)}$

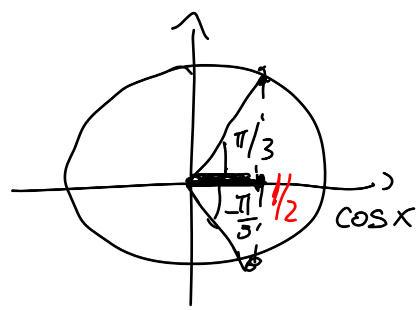
b) During the first revolution, at what times is your vertical velocity equal to π ft/s?

$y'(t) = \pi$

$y'(t) = 60\cos\left(\frac{\pi}{30}(t-15)\right)\cdot\frac{\pi}{30}$ (Chain Rule!)

$y'(t) = 2\cancel{\pi}\cos\left(\frac{\pi}{30}(t-15)\right) = \cancel{\pi}$

$\cos\left(\frac{\pi}{30}(t-15)\right) = \frac{1}{2}$



$\frac{\pi}{30}(t-15) = \frac{\pi}{3}$ & $\frac{\pi}{30}(t-15) = -\frac{\pi}{3}$

$t-15=10$ $\quad$ $t-15=-10$

at $\boxed{t=25\text{ sec}}$ and at $\boxed{t=5\text{ sec}}$

Recall, your coordinates at time t seconds are:

$$x(t) = 60\cos\left(\frac{\pi}{30}(t-15)\right), \quad y(t) = 60\sin\left(\frac{\pi}{30}(t-15)\right) + 63.$$

c) What is your maximal vertical acceleration? (include units)

$$y''(t) = \left[2\pi\cos\left(\frac{\pi}{30}(t-15)\right)\right]' = 2\pi\left(-\sin\left(\frac{\pi}{30}(t-15)\right)\right)\frac{\pi}{30}$$

$$= -\frac{\pi^2}{15}\sin\left(\frac{\pi}{30}(t-15)\right)$$ is maximal when $\sin\left(\frac{\pi}{30}(t-15)\right) = -1$

and its max value is

$$\boxed{\max y''(t) = \pi^2/15 \ \ ft/s^2}$$

d) You turn on a flashlight, and you point the light beam in a direction tangent to your trajectory.

Find the equation of the beam of light at $t = 25$ seconds. (Hint: $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$)

need slope & coordinates at $t = 25$ seconds

$$(x(25), y(25)) = \left(60\cos\frac{\pi}{3}, 60\sin\frac{\pi}{3} + 63\right)$$
$$= (30, 30\sqrt{3} + 63)$$

$$\left.\frac{dy}{dx}\right|_{t=25} = \left.\frac{dy/dt}{dx/dt}\right|_{t=25} =$$

$$= \frac{\cancel{2\pi}\cos\pi/3}{-\cancel{2\pi}\sin\pi/3} = -\frac{1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$$

Line equation: $\boxed{y - (30\sqrt{3} + 63) = -\frac{1}{\sqrt{3}}(x - 30)}$

Simplified: $\boxed{y = -1/\sqrt{3}\, x + 40\sqrt{3} + 63}$