Worksheet

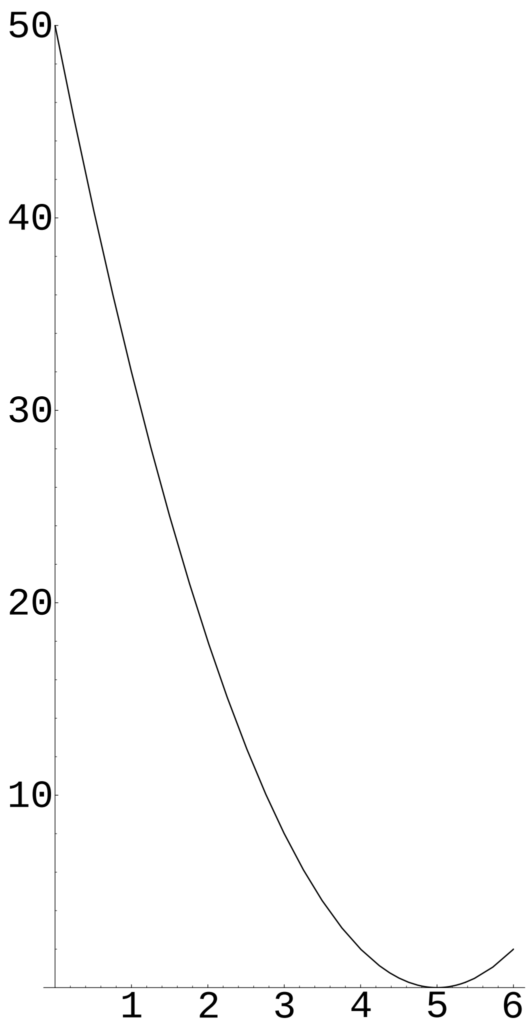
Average and Instantaneous Velocity Math 124

Introduction

In this worksheet, we introduce what are called the *average and instantaneous velocity* in the context of a specific physical problem: A golf ball is hit toward the cup from a distance of 50 feet. Assume the distance from the ball to the cup at time t seconds is given by the function

$$d(t) = 50 - 20t + 2t^2.$$

The graph of $y = d(t)$ appears below.



1. Does the ball reach the cup? If so, when? (Answer this question two ways: by using algebra, and by reading the graph.)

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2. (a) Plot and label these points on the graph:

$$\begin{aligned} P &= (2, d(2)) \\ Q_2 &= (4, d(4)) \\ Q_1 &= (3, d(3)) \\ Q_{0.5} &= (2.5, d(2.5)) \\ Q_{0.01} &= (2.01, d(2.01)) \end{aligned}$$

(b) Sketch the line through P and Q_2
Sketch the line through P and Q_1
Sketch the line through P and $Q_{0.5}$
Sketch the line through P and $Q_{0.01}$

(c) Compute the slopes of the lines in (b).

(d) We define the average velocity as follows: average velocity$=\frac{\text{distance traveled}}{\text{time elapsed}}$.
Explain why the slope of the line $\overline{PQ_2}=$ the average velocity from $t=2$ seconds to $t=4$ seconds.

(e) Find the average velocity over the following time intervals:

$t=2$ seconds to $t=4$ seconds:

$t=2$ seconds to $t=3$ seconds:

$t=2$ seconds to $t=2.5$ seconds:

$t=2$ seconds to $t=2.01$ seconds:

(f) The average velocities in (e) approach a number as the time interval gets smaller and smaller. Guess this number.

3. Let h be a small constant positive number and define $Q_h = (2+h, d(2+h))$. Compute the slope of the secant line connecting P and Q_h by simplifying:

$$slope = \frac{(\text{y coordinate}\, Q_h) - (\text{y coordinate}\, P)}{(\text{t coordinate}\, Q_h) - (\text{t coordinate}\, P)}$$

so there is no h in the denominator. This slope = the average velocity on the time interval $t = 2$ seconds to $t = 2 + h$ seconds.

4. What number do you get when you plug $h = 0$ into the simplified expression in problem 3 above? This is called the instantaneous velocity at $t = 2$ seconds.

5. Draw a line through P with slope equal to the number computed in the previous step. How would you describe this line relative to the graph?