Worksheet- Week #2

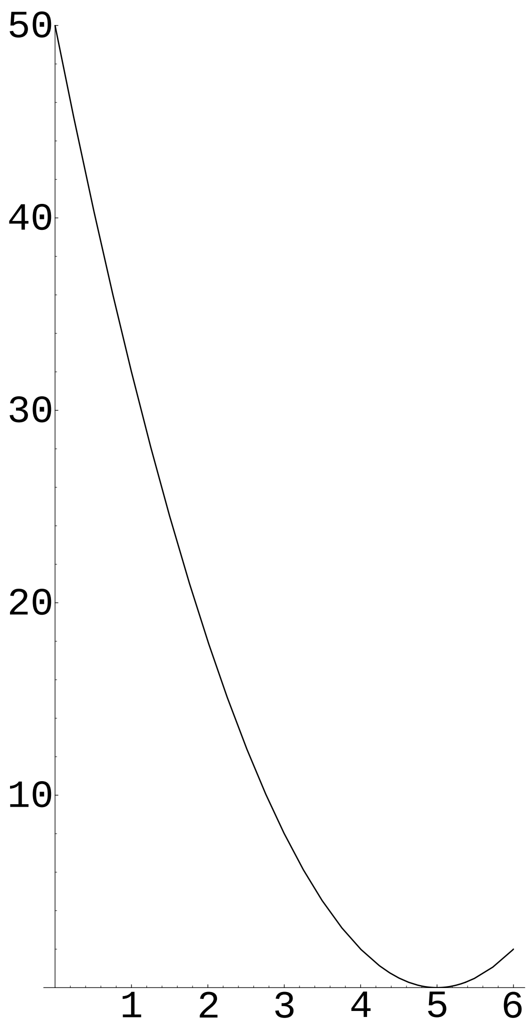
Average and Instantaneous Velocity Math 124

Introduction

In this worksheet, we introduce what are called the *average and instantaneous velocity* in the context of a specific physical problem: A golf ball is hit toward the cup from a distance of 50 feet. Assume the distance from the ball to the cup at time t seconds is given by the function

$$d(t) = 50 - 20t + 2t^2.$$

The graph of $y = d(t)$ appears below.



1. Does the ball reach the cup? If so, when? (Answer this question two ways: by using algebra, and by reading the graph.)

2. (a) Plot and label these points on the graph:

$$\begin{aligned} P &= (2, d(2)) \\ Q_2 &= (4, d(4)) \\ Q_1 &= (3, d(3)) \\ Q_{0.5} &= (2.5, d(2.5)) \\ Q_{0.01} &= (2.01, d(2.01)) \end{aligned}$$

(b) Sketch the line through P and Q_2
Sketch the line through P and Q_1
Sketch the line through P and $Q_{0.5}$
Sketch the line through P and $Q_{0.01}$

(c) Compute the slopes of the lines in (b).

(d) We define the average velocity as follows: average velocity$=\frac{\text{distance traveled}}{\text{time elapsed}}$.
Explain why the slope of the line $\overline{PQ_2}=$ the average velocity from $t = 2$ seconds to $t = 4$ seconds.

(e) Find the average velocity over the following time intervals:

$t = 2$ seconds to $t = 4$ seconds:

$t = 2$ seconds to $t = 3$ seconds:

$t = 2$ seconds to $t = 2.5$ seconds:

$t = 2$ seconds to $t = 2.01$ seconds:

(f) The average velocities in (e) approach a number as the time interval gets smaller and smaller. Guess this number.

3. Let h be a small constant positive number and define $Q_h = (2+h, d(2+h))$. Compute the slope of the secant line connecting P and Q_h by simplifying:

$$slope = \frac{(\text{y coordinate}\, Q_h) - (\text{y coordinate}\, P)}{(\text{t coordinate}\, Q_h) - (\text{t coordinate}\, P)}$$

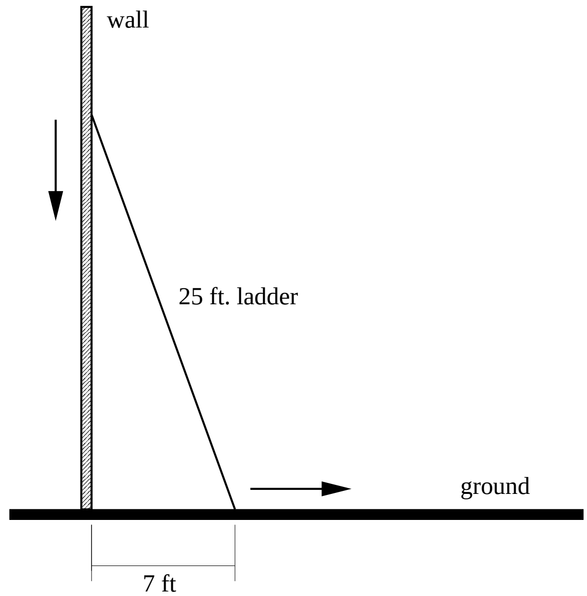
so there is no h in the denominator. This slope = the average velocity on the time interval $t = 2$ seconds to $t = 2 + h$ seconds.

4. What number do you get when you plug $h = 0$ into the simplified expression in problem 3 above? This is called the instantaneous velocity at $t = 2$ seconds.

5. Draw a line through P with slope equal to the number computed in the previous step. How would you describe this line relative to the graph?

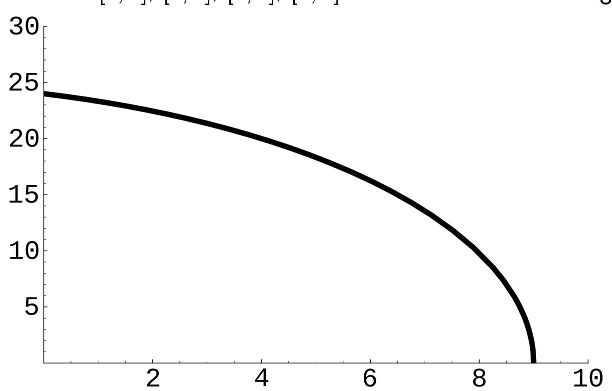
If your group has finished, start working on this problem from homework #2:

A ladder 25 feet long is leaning against the wall of a building. Initially, the foot of the ladder is 7 feet from the wall. The foot of the ladder begins to slide at a rate of 2 ft/sec, causing the top of the ladder to slide down the wall. The location of the foot of the ladder at time t seconds is $(7+2t, 0)$.



(a) The location of the top of the ladder will be $(0, y(t))$, for some function $y(t)$. Find the formula for $y(t)$. What is the domain of t values?

(b) The graph of the function $y(t)$ is given below. Compute the average velocity of the top of the ladder on these time intervals: $[0,2]$, $[2,4]$, $[6,8]$, $[8,9]$. Indicate what the average velocity is telling you in terms of the picture.



$[0,2]$: $v_{avg} =$

$[2,4]$: $v_{avg} =$

$[6,8]$: $v_{avg} =$

$[8,9]$: $v_{avg} =$

(c) The foot of the ladder is moving at a constant rate; how about the top of the ladder?