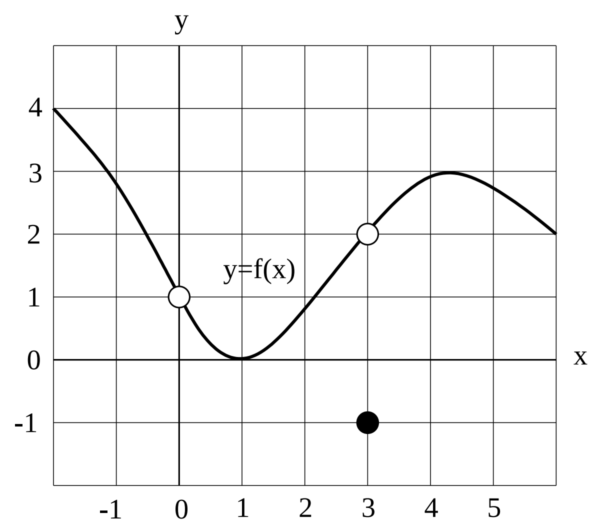
Worksheet

Continuity and Limits Math 124

Introduction

This worksheet discusses a method of computing limits for some special functions.

1. Use the graph of the function f at right to answer the following questions. Assume this is the entire graph of f.
   (a) What is the domain of f?



   (b) Find $\lim_{x \to 3} f(x)$.

   (c) Find $\lim_{x \to 0} f(x)$.

   (d) What is $f(3)$?

DEFINITION: A function g is called **continuous at** a if two things are true:

(i) a is a point in the domain of g and (ii) $\lim_{x \to a} g(x) = g(a)$.

A function is just called **continuous** if it is continuous at every point in its domain.

2. Answer the following questions for the function f whose graph is above. Explain your answers.
   (a) Is f continuous at 3?
   (b) Is f continuous at 0?
   (c) Is f continuous at 1?
   (d) Is f continuous?

Not every function is continuous, but many are. It's often important to know ahead of time that a function we want to work with is continuous, because that makes limit calculations easy. The following kinds of functions are *always* continuous:

- constant functions
- linear functions
- polynomials
- trigonometric functions
- exponential functions

There are many other kinds of continuous functions; some will be explored later in the course. You can also combine continuous functions to make new continuous functions. For example, you can add, multiply or compose continuous functions and be guaranteed to get a continuous function. Division, on the other hand, sometimes creates complications, as we'll see below.

3. Find $\lim_{x \to 2} 3x^2 - 2x + 4.$ Explain your solution.

4. Can you find $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ by plugging in $x = 1$? Explain.

Sometimes we can find a limit of a function where it isn't continuous or defined by first simplifying the function, so that it resembles a continuous function.

5. Simplify the function in problem number 4, and find the limit of the resulting expression as $x \to 1$. Why can you say that this is the same as the limit you were asked to find in problem number 4?

6. Let $e(x) = 125x^2 - 6.25x^3$.

(a) Simplify the expression $\frac{e(5+h)-e(5)}{(5+h)-(5)}$ as much as possible; there should not be an h in the denominator of any fraction in the simplified expression.

(b) Is $\frac{e(5+h)-e(5)}{(5+h)-(5)}$ continuous at $h = 0$? Why?
Is the simplified expression in part (a) continuous at $h = 0$? Why?

(c) Calculate $\lim_{h \to 0} \dfrac{e(5+h)-e(5)}{(5+h)-(5)}$ using the approach outlined in problem 5.

7. Let $e(x) = 125x^2 - 6.25x^3$, as in problem 6. Let x be an unknown constant.

(a) Simplify the expression $\frac{e(x+h)-e(x)}{(x+h)-(x)}$ as much as possible; there should not be an h in the denominator of any fraction in the simplified expression.

(b) Is $\frac{e(x+h)-e(x)}{(x+h)-(x)}$ continuous at $h = 0$? Why?
Is the simplified expression in part (a) continuous at $h = 0$? Why?

(c) Calculate $\lim_{h \to 0} \dfrac{e(x+h) - e(x)}{(x+h) - (x)}$ using the approach outlined in problem 5.