Worksheet-Week #7

Sinusoidal functions and the Piston Problem Math 124

Introduction. In this worksheet, we will begin by investigating derivatives of sinusoidal functions. We then model the motion of a piston and study its velocity and acceleration.

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1. **Derivatives of Sinusoidal Functions** Let $f(t) = 4\sin(\pi t)$. Calculate each derivative and put it into standard sinusoidal form. You will need to use the identity: $\cos(\theta) = \sin(\theta + \frac{\pi}{2})$.

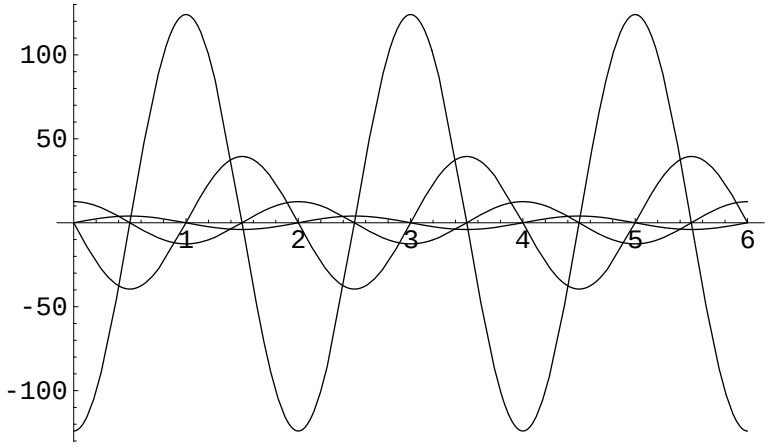
   (a) $f'(t) =$

   (b) $f''(t) =$

   (c) $f'''(t) =$

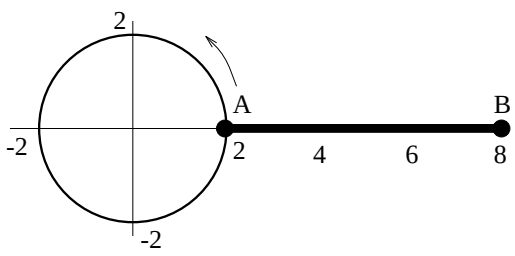
   (d) $f^{(4)}(t) =$

   (e) Below are the graphs of $f(t)$ and the first three derivatives; identify each curve.

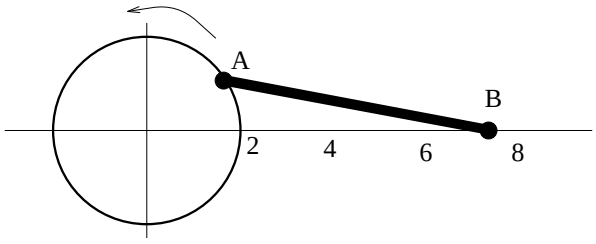


   (f) True or False: The derivative of a sinuosidal function is a sinusodial function.

The Piston A six foot long rod is attached at one end A to a point on a wheel of radius 2 feet, centered at the origin. The other end B is free to move back and forth along the x-axis. The point A is at $(2, 0)$ at time $t = 0$, and the wheel rotates counterclockwise at constant speed with an angular speed of 3/2 revolutions per minute.



time t=0



time t > 0

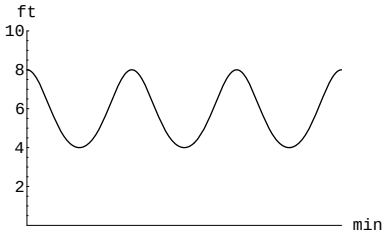
2. Let $x(t)$ be the x-coordinate of the point B as a function of time t minutes; what is the formula for $x(t)$?

3. Calculate $x'(t)$.

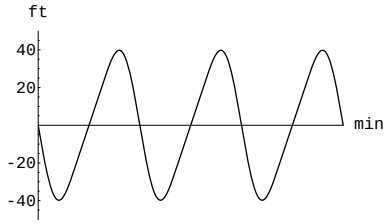
5. **Use calculus** to determine when the velocity of B is zero. What is the picture of the piston when this happens?

4. The graphs of $x(t)$, $x'(t)$ and $x''(t)$ on the domain $0 \leq t \leq 1$ minute are below. The graph of $x(t)$ looks like the graph of a sinusoidal function. Is $x(t)$ a sinusoidal function? Is $x(t)$ a periodic function? How can you tell? (Hint: Go back to 1.)

$x(t)$



$x'(t)$



$x''(t)$