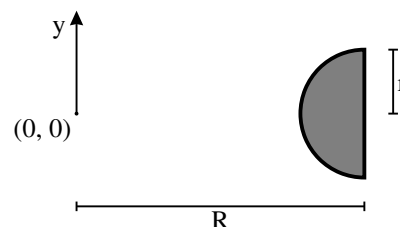


- 1 Stewart, section 7.1: #1, 5, 7, 15, 19, 27, 33, 35, 62, 63, 65
- 2 Stewart, section 7.2: #5, 9, 13, 19, 27, 57, 65
- 3 Stewart, section 7.3: #3, 5, 7, 25, 26, 27, 28, 33, 42

4 A doughnut has been partially eaten by a meticulous person so that the portion remaining is given by rotating the half circular region shown about the y -axis. What proportion of the doughnut remains, assuming that the doughnut was the volume of revolution of the full circle?



- 5 A pair of functions f and g are said to be *orthogonal on the interval* $[a, b]$ if $\int_a^b f(x)g(x) dx = 0$. A family of functions is said to be *orthogonal* on $[a, b]$ if each distinct pair of functions $f \neq g$ in the family is orthogonal on $[a, b]$.

Show that the family of trigonometric functions

$$\{\sin(x), \cos(x), \sin(2x), \cos(2x), \dots, \sin(nx), \cos(nx), \dots\}$$

is orthogonal on the interval $[0, 2\pi]$. (You need to show that the pairs $\sin(Mx), \sin(Nx)$ and $\cos(Mx), \cos(Nx)$ are orthogonal for all positive integers $M \neq N$, and that the pair $\sin(Mx), \cos(Nx)$ is orthogonal for all positive integers M and N .)

This result about the orthogonality of the sine and cosine functions is the basis for an important area of mathematics called Fourier series.

- 6 a) Determine the average values of x, x^2, x^3 , and x^4 on the interval $[1, 2]$.
- b) Determine the average values of $\sin(x), \sin^2(x), \sin^3(x)$, and $\sin^4(x)$ on the interval $[0, \pi]$.
- c) Explain geometrically why the average values in a) increased, while those in b) decreased.