1 Stewart, section 9.1: #1, 2, 3, 5, 7, 9, 14

2 Stewart, section 9.3: #1, 2, 3, 9, 11, 12, 13, 15, 39, 40, 41

3 Stewart, section 3.8: #3, 10, 11, 13, 14

4 Suppose you drop a stone of mass m from a great height in the earth's atmosphere, and the only forces acting on the stone are the earth's gravitational attraction ($g \approx 9.8 m/s^2$) and a retarding force due to air resistance, which is proportional to the velocity v. Then, since $F = ma$ and $a = \frac{dv}{dt}$, we have the differential equation $m\frac{dv}{dt} = mg - kv$ (where k is some constant).

a) Find v as a function of time t.

b) Calculate $\lim_{t\to\infty} v(t)$, the terminal velocity of the stone.

5 Experimenting, you have found that 12 ounces of 180° F coffee in your favorite cup will take 20 minutes to cool to a drinking temperature of 110° F in a 70° F room.

Assume that when you add cream to the coffee, the two liquids are mixed together instantly, and the temperature of the mixture instantly becomes the weighted average of the temperature of the coffee and of the cream (weighted by the number of ounces of each fluid). Also assume that the cooling constant of the liquid (the constant k in the equations on p. 617 for Newton's law of cooling) doesn't change when you add the cream.

a) If you add 2 ounces of 40° F cream to the 180° cup, how long will it take for the mixture to reach drinking temperature?

b) If you let the 12 ounces of 180° coffee cool for 5 minutes before adding 2 ounces of 40° cream, how long do you have to wait before the mixture reaches drinking temperature?

c) In order to reach drinking temperature as quickly as possible, should you add the cream immediately, or wait a while?

6 This problem models pollution effects in the Great Lakes. We assume pollutants are flowing into a lake at a constant rate I, and that water is flowing out at a constant rate F. We also assume that the pollutants are uniformly distributed throughout the lake. If $c(t)$ denotes the concentration of pollutants at time t, then it satisfies the differential equation

$$c'(t) = -\frac{F}{V}c + \frac{I}{V},$$

where V is the volume of the lake. We assume that rain and streams flowing into the lake keep the volume of water in the lake constant.

a) Solve this differential equation assuming $c(0) = c_0$.

b) Compute $c_\infty = \lim_{t\to\infty} c(t)$.

c) For Lake Erie, $V = 458$ km^3 and $F = 175$ km^3/year. Suppose that one day its pollutant concentration is c_1 and that all pollution suddenly stopped. Determine the number of years it would then take for pollution levels to drop to $c_1/10$. (Hint: Use the solution to (a) with $I = 0$ and $c_0 = c_1$.)

d) Rework the previous problem for Lake Superior where $V = 12221$ km^3 and $F = 65.2$ km^3/year.

7 A person borrows \$10,000 and repays the loan at the rate of \$2400/year. The lender charges interest of 10%/year. Assuming that payments are made continuously and interest is compounded continuously (a pretty good approximation to reality for long-term loans), the amount of money owed t years after the loan is made, $M(t)$, then satisfies

$$\frac{dM}{dt} = \tfrac{1}{10}M - 2400, \qquad M(0) = 10000.$$

a) Solve this ODE for $M(t)$.

b) (b) How long does it take to pay off the loan? That is, at what time t is $M(t) = 0$?